Section 8.1: INTRODUCTION TO FUNCTIONS

When you are done with your homework you should be able to…

- π Find the domain and range of a relation
- π Determine whether a relation is a function
- π Evaluate a function

WARM-UP:

Evaluate $y=-x^2-22x+5$ at $x=-3$.

DEFINITION OF A RELATION

A ________________ is any _______ of ordered pairs. The set of all ________ components of the ____________ pairs is called the ____________ of the relation and the set of all second components is called the ____________ of the ____________.

Example 1: Find the domain and range of the relation.

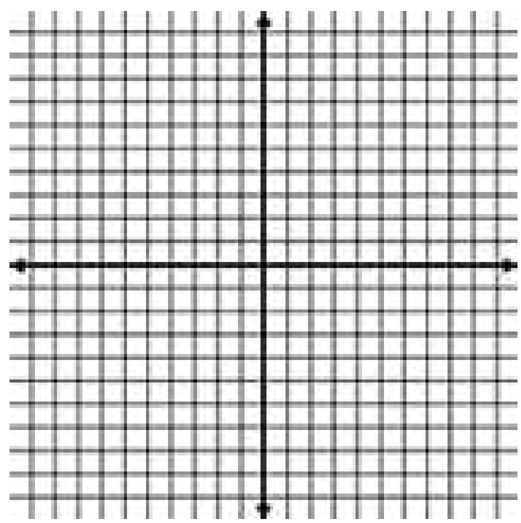
VEHICLE	NUMBER OF WHEELS
CAR	4
MOTORCYCLE	2
BOAT	0

DEFINITION OF A FUNCTION

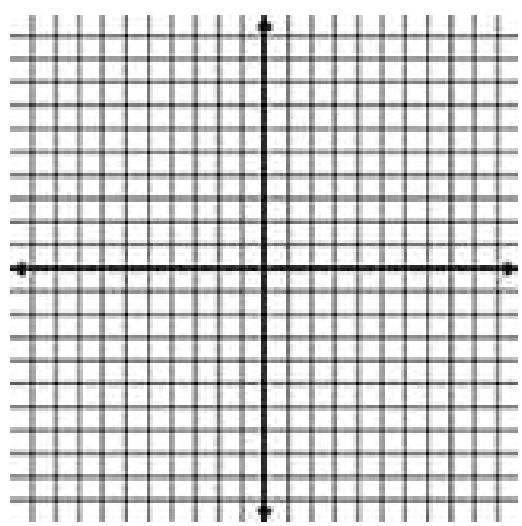
A ________________ is a __________________ from a first set, called the _______________, to a second set, called the _______________, such that each _______________ in the ______________ corresponds to ___________ ______ element in the ______________.

Example 2: Determine whether each relation represents a function. Then identify the domain and range.

a. $\{(-6,1),\ (-1,1),\ (0,1),\ (1,1),\ (2,1)\}$



b. $\{(3,3),\ (-2,0),\ (4,0),\ (-2,-5)\}$



FUNCTIONS AS EQUATIONS AND FUNCTION NOTATION

Functions are often given in terms of _______________ rather than as _______

of ________________ ______________. Consider the equation below, which describes the position of an object, in feet, dropped from a height of 500 feet after x seconds.

$$y = -16x^2 + 500$$

The variable ___ is a ______________ of the variable ____. For each value of x, there is one and only one value of ____. The variable x is called the _______________ variable because it can be _______________ any value from the _______________. The variable y is called the _______________ variable because its value ______________ on x. When an ________________ represents a ________________, the function is often named by a letter such as $f,\ g,\ h,\ F,\ G,$ or H. Any letter can be used to name a function. The domain is the _____ of the function's ______________ and the range is the _____ of the function's _______________. If we name our function ____, the input is represented by ____, and the output is represented by _____. The notation _____ is read " ___ of ___" or "___ at ___. So we may rewrite $y = -16x^2 + 500$ as _____________________. Now let's evaluate our function after 1 second:

Example 3: Find the indicated function values for $f(x) = (-x)^3 - x^2 - x + 10$.

a. $f(0)$

b. $f(2)$

c. $f(-2)$

d. $f(1) + f(-1)$

Example 4: Find the indicated function and domain values using the table below.

a. $h(-2)$

b. $h(1)$

c. For what values of x is $h(x) = 1$?

x	$h(x)$
-2	2
-1	1
0	0
1	1
2	2

Section 8.2: GRAPHS OF FUNCTIONS

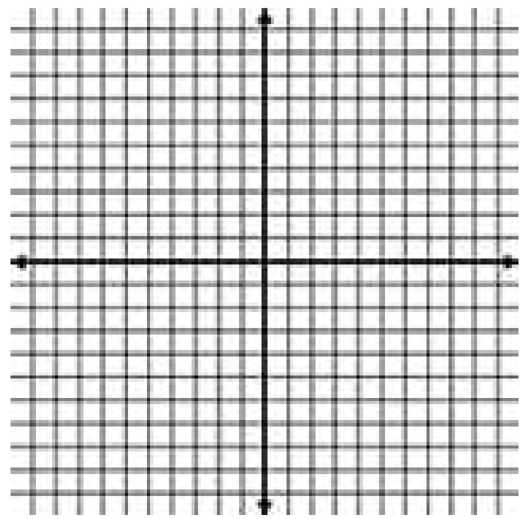
When you are done with your homework you should be able to...

- π Use the vertical line test to identify functions
- π Obtain information about a function from its graph
- π Review interval notation
- π Identify the domain and range of a function from its graph

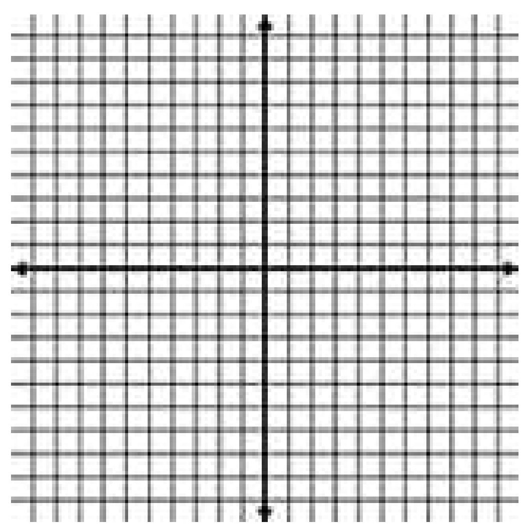
WARM-UP:

Graph the following equations by plotting points.

a. $y = x^2$



b. $y = 3x - 1$

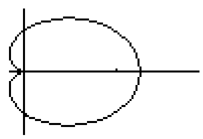


THE VERTICAL LINE TEST FOR FUNCTIONS

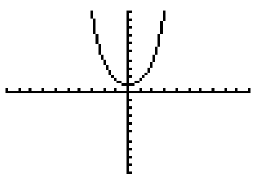
If any vertical line _________________ a graph in more than __________ point, the graph _________ __________ define ____ as a function of ____.

Example 1: Determine whether the graph is that of a function.

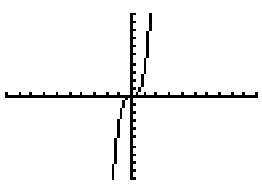
a.



b.



c.



OBTAINING INFORMATION FROM GRAPHS

You can obtain information about a function from its graph. At the right or left of a graph, you will often find ___________ dots, ___________ dots, or __________.

π A closed dot indicates that the graph does not ___________ beyond this point and the ____________ belongs to the ______________

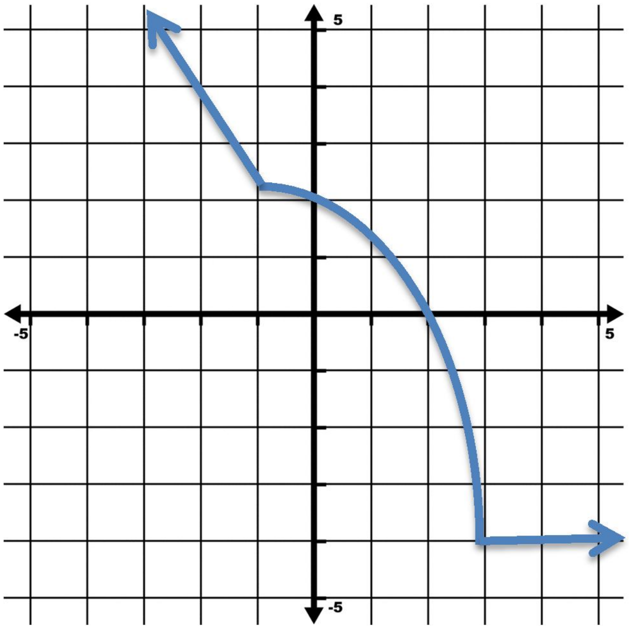
π An open dot indicates that the graph does not ___________ beyond this point and the ____________ DOES NOT belong to the ______________ π An arrow indicates that the graph extends ________________ in the direction in which the arrow ________________

REVIEWING INTERVAL NOTATION

INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
(a,b)		x
$[a,b]$		x
$[a,b)$		x
$(a,b]$		x
(a,∞)		x
$[a,\infty)$		x
$(-\infty,b)$		x
$(-\infty,b]$		x
$(-\infty,\infty)$		x

Example 2: Use the graph of f to determine each of the following.

f



a. $f(0)$

b. $f(-2)$

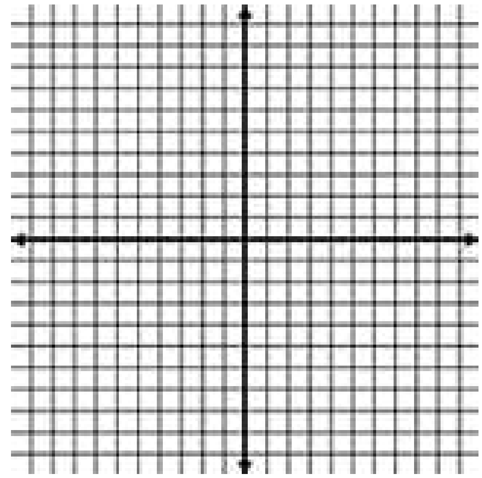
c. For what value of x is $f(x)=3$?

d. The domain of f

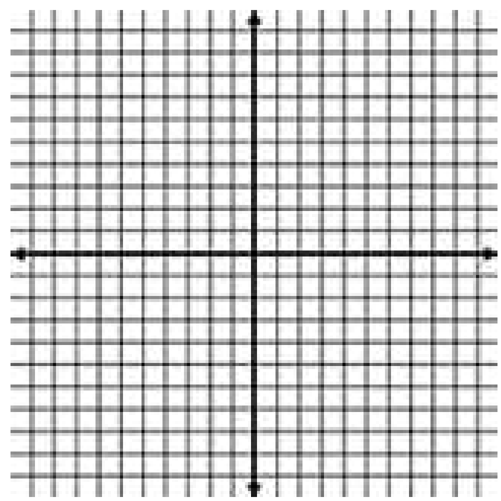
e. The range of f

Example 3: Graph the following functions by plotting points and identify the domain and range.

a. $f(x) = -x - 2$



b. $H(x) = x^2 + 1$



Section 8.3: THE ALGEBRA OF FUNCTIONS

When you are done with your homework you should be able to…

- π Find the domain of a function
- π Use the algebra of functions to combine functions and determine domains

WARM-UP:

Find the following function values for $f(x)=\sqrt{x}$

a. $f(4)$

b. $f(0)$

c. $f(196)$

FINDING A FUNCTION'S DOMAIN

If a function f does not model data or verbal conditions, its domain is the

_______________ set of __________ numbers for which the value of $f(x)$ is a real

number. _________________ from a function's _______________ real numbers

that cause _________________ by _______________ and real numbers that result

in a _______________ root of a ___________________ number.

Example 1: Find the domain of each of the following functions.

a. $f(x)=\sqrt{x-1}$

b. $g(x)=\dfrac{4-x}{1-x^2}$

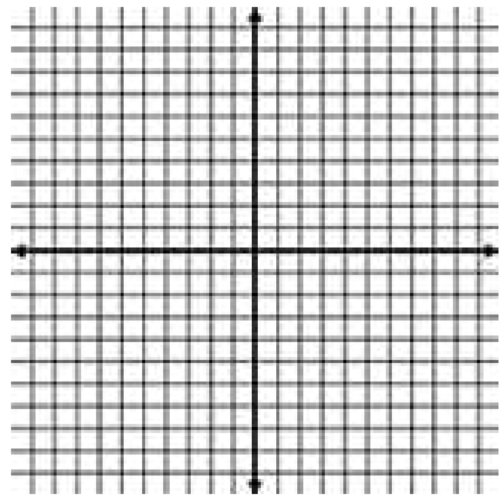
c. $h(t)=3t+5$

THE ALGEBRA OF FUNCTIONS

Consider the following two functions:

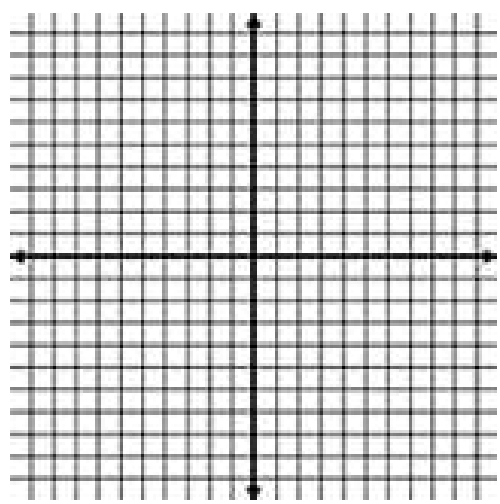
$f(x) = -x$ and $g(x) = 3x - 5$

Let's graph these two functions on the same coordinate plane.



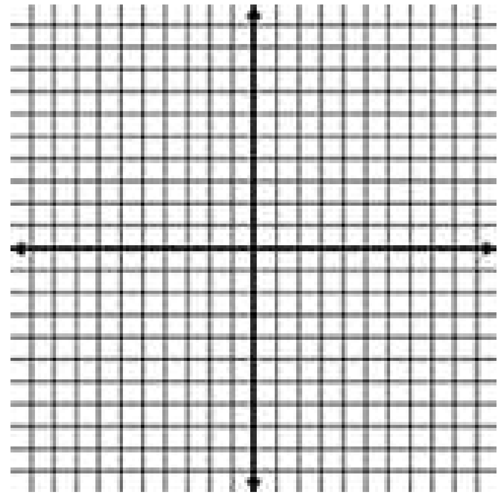
Now find and graph the sum of f and g.

$(f+g)(x) =$



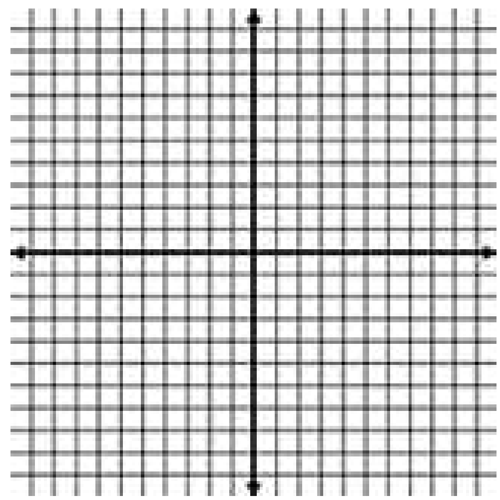
Now find and graph the difference of f and g.

$(f-g)(x)=$



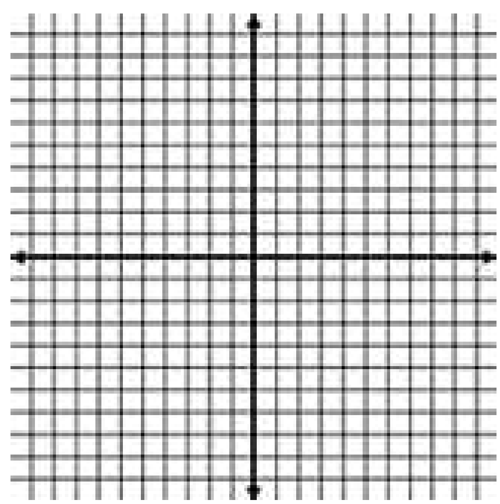
Now find and graph the product of f and g.

$(fg)(x)=$



Now find and graph the quotient of f and g.

$\left(\frac{f}{g}\right)(x)=$



THE ALGEBRA OF FUNCTIONS: SUM, DIFFERENCE, PRODUCT, AND QUOTIENT OF FUNCTIONS

Let f and g be two functions. The ______ $f+g$, the _____________ $f-g$, the ____________ fg, and the ____________ $\frac{f}{g}$ are ____________ whose domains are the set of all real numbers _______________ to the domains of f and g, defined as follows:

1. Sum: ______________________________
2. Difference: _________________________
3. Product: ____________________________
4. Quotient: ___________________________, provided _______________

Example 2: Let $f(x)=x^2+4x$ and $g(x)=2-x$. Find the following:

a. $(f+g)(x)$

b. $(f+g)(4)$

c. $f(-3)+g(-3)$

d. $(fg)(x)$

e. $(fg)(3)$

f. The domain of $\left(\frac{f}{g}\right)(x)$

Section 8.4: COMPOSITE AND INVERSE FUNCTIONS

When you are done with your homework you should be able to…

- π Form composite functions
- π Verify inverse functions
- π Find the inverse of a function
- π Use the horizontal line test to determine if a function has an inverse function
- π Use the graph of a one-to-one function to graph its inverse function

WARM-UP:

Find the domain and range of the function $\{(-1,0),(0,1),(1,2),(2,3)\}$:

THE COMPOSITION OF FUNCTIONS

The composition of the function ____ with ____ is denoted by __________ and is defined by the equation

The domain of the ______________ function __________ is the set of all ____ such that

1. ____ is in the domain of ____ and

2. ______ is in the domain of _____.

Example 1: Given $f(x) = -x^2 + 8$ and $g(x) = 6x - 1$, find each of the following composite functions.

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

DEFINITION OF THE INVERSE OF A FUNCTION

Let f and g be two functions such that

__________________ for every _____ in the domain of ____

and

__________________ for every _____ in the domain of ____.

The function _____ is the _____________ of the function _____ and is denoted

by ______ (read "f-inverse"). Thus _______________ and ______________.

The _______________ of ____ is equal to the ____________ of ______ and

vice versa.

Example 2: Show that each function is the inverse of the other.

$f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$

FINDING THE INVERSE OF A FUNCTION

The equation of the inverse of a function f can be found as follows:

1. Replace ________ with ______ in the equation for ________.

2. Interchange _____ and _____.

3. Solve for _____. If this equation does not define ___ as a function of ___, the function _____ doe not have an _____________ function and this procedure ends. If this equation does define ___ as a function of ___, the function ___ has an inverse function.

4. If ___ has an inverse function, replace ___ in step 3 with ________. We can verify our result by showing that __________________ and ________________.

Example 3: Find an equation for $f^{-1}(x)$, the inverse function.

a. $f(x) = 4x$

b. $f(x) = \dfrac{2x-3}{x+1}$

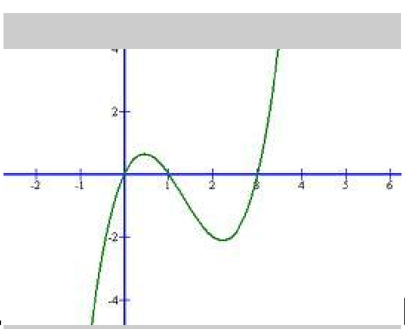
THE HORIZONTAL LINE TEST FOR INVERSE FUNCTIONS

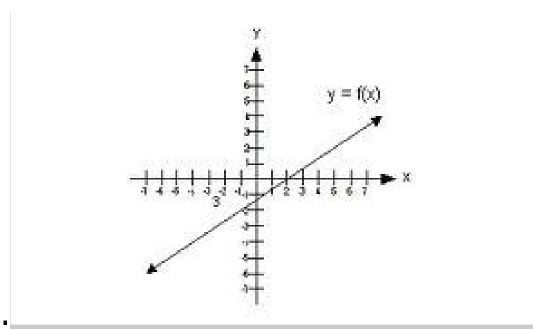
A function f has an inverse that is a function ________, if there is no

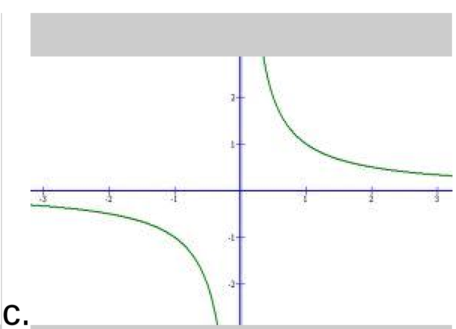
__________________ line that intersects the graph of the function ______ at

more than __________ point.

Example 4: Which of the following graphs represent functions that have inverse functions?

a. 

b. 

c. 

GRAPHS OF A FUNCTION AND ITS INVERSE FUNCTION

There is a ____________________ between the graph of a one-to-one function

____ and its inverse _______. Because inverse functions have ordered pairs with

the coordinates _________________, if the point __________ is on the graph

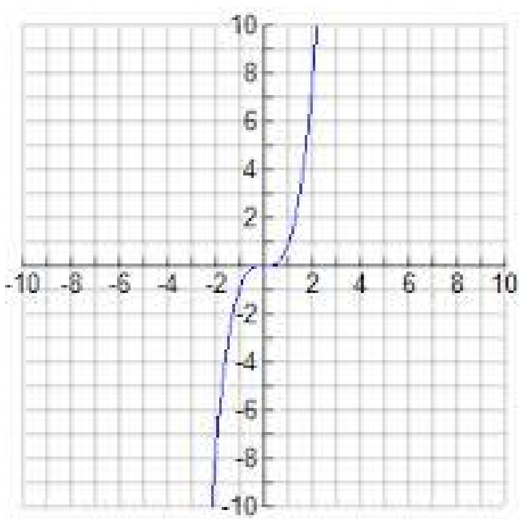
of _____, the point ____________ is on the graph of _________. The points

______ and ______ are _________________ with respect to the line _______.

Therefore, the graph of _______ is a ___________________ of the graph

of _____ about the line __________.

Example 5: Use the graph of f below to draw the graph of its inverse function.



Section 9.1: REVIEWING LINEAR INEQUALITIES AND USING INEQUALITIES IN BUSINESS APPLICATIONS

When you are done with your homework you should be able to...

- π Review how to solve linear inequalities
- π Use linear inequalities to solve problems involving revenue, cost, and profit

WARM-UP:

Solve.

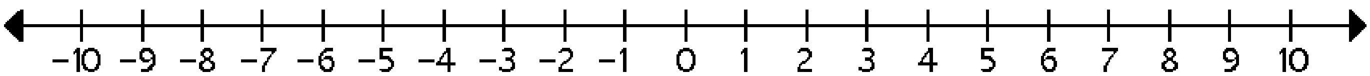
$5-8(12-5x)=x$

SOLVING A LINEAR INEQUALITY

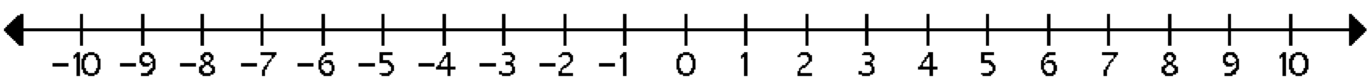
1. Simplify the ________________ expression on each side.
2. Use the ________________ property of inequality to collect all the ________________ terms on one side and the _________________ terms on the other side.
3. Use the __________________ property of inequality to _______________ the variable and solve. Change the _______________ of the inequality when multiplying or dividing both sides by a _______________ number.
4. Express the solution set in _________________ notation and graph the solution set on a _______________ line.

Example 1: Solve and graph the solution on a number line.

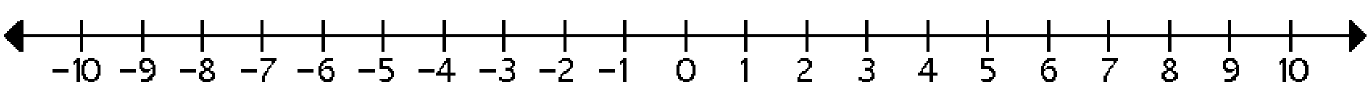
a. $2x+5<17$



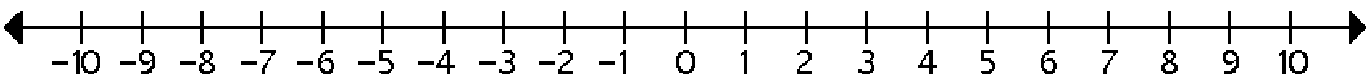
b. $-4(x+2)\geq 3x+20$



c. $\frac{4x-3}{6}+2>\frac{2x-1}{12}$



Example 2: Let $f(x)=\frac{2}{5}(10x-15)+9$ and let $g(x)=\frac{3}{8}(16-8x)-7$. Find all values of x for which $g(x)\le f(x)$.



FUNCTIONS OF BUSINESS AND LINEAR INEQUALITIES

For any business, the ____________________ function, ___________, is the money generated by selling ____ units of the product:

The _____________ function, ___________, is the ___________ of producing _____ units of the product:

The term on the right, ________, represents _______________ cost because it ____________ based on the number of units _____________.

REVENUE, COST, AND PROFIT FUNCTIONS

A company produces and sells ____ units of a product.

REVENUE FUNCTION:

COST FUNCTION:

PROFIT FUNCTION:

APPLICATIONS

1. A company that manufactures bicycles has a fixed cost of $100,000. It costs $100 to produce each bicycle. The selling price is $300 per bike. Let *x* represent the number of bicycles produced and sold.
   a. Write the cost function, *C.*

   b. Write the revenue function, *R.*

c. Write the profit function, *P.*

d. More than how many units must be produced and sold for the business to make money?

2. You invested $30,000 and started a business writing greeting cards. Supplies cost $0.02 per card and you are selling each card for $0.50. Let x represent the number of cards produced and sold.
   a. Write the cost function, *C.*

   b. Write the revenue function, *R.*

   c. Write the profit function, *P.*

   d. More than how many units must be produced and sold for the business to make money?

Section 9.2: COMPOUND INEQUALITIES

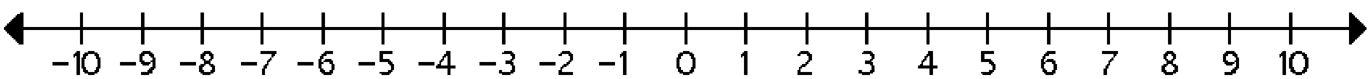
When you are done with your homework you should be able to…

- π Find the intersection of two sets
- π Solve compound inequalities involving *and*
- π Find the union of two sets
- π Solve compound inequalities involving *or*

WARM-UP:

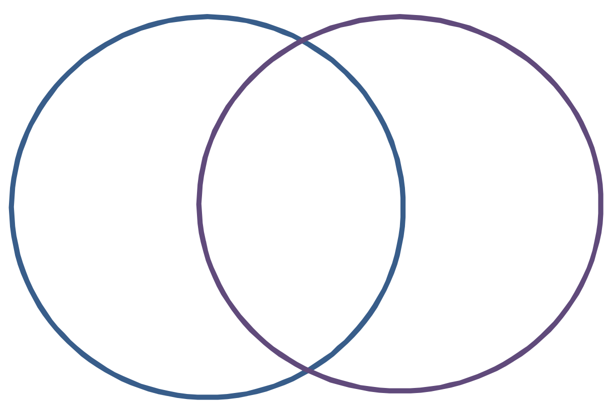
Solve and graph the solutions of the inequality.

$-6x+7>-(x-12)$



Consider the following situation:

Shannon has 2 dogs and 2 cats. Jill has 1 dog and no cats. Nicole has 1 dog and 2 cats. Let C represents the set of these people who have cats. Let D represent the set of these people who have dogs.



COMPOUND INEQUALITIES INVOLVING *AND*

If _____ and _____ are sets, we can form a new set consisting of all

____________________ that are in ___________ *A* and *B*. This is called the

____________________ of the two sets.

DEFINITION OF THE INTERSECTION OF SETS

The ______________________ of sets ____ and ____, written ____________, is the set of elements __________________ to ____________ set _____ **and** set _____. This definition can be expressed in set-builder notation as follows:

Example 1: Find the intersection of the sets.

a. $\{1,3,7\}\cap\{2,3,8\}$

b. $\{1,2,3,4,5\}\cap\{2,4,6\}$

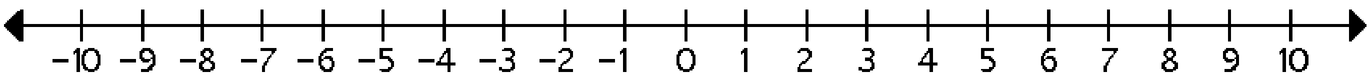
c. $\{-4,-3,-1\}\cap\{-2,3,4\}$

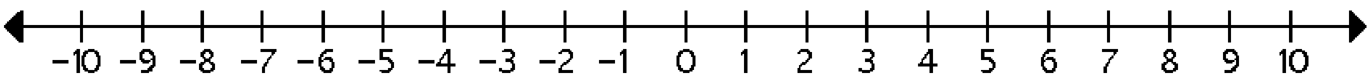
SOLVING COMPOUND INEQUALITIES INVOLVING *AND*

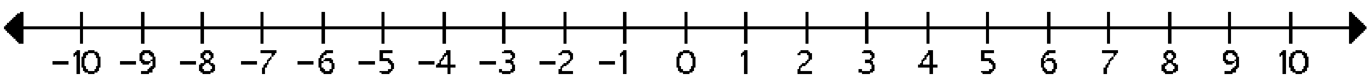
1. Solve each inequality ____________________.
2. Graph the solution set to ___________ inequality on a number line and take the _____________________ of these solution sets. This is where the sets ____________________.

Example 2: Solve each compound inequality. Use graphs to show the solution set to each of the two given inequalities, as well as a third graph that shows the solution set of the compound inequality. Except for the empty set, express the solution set in interval notation.

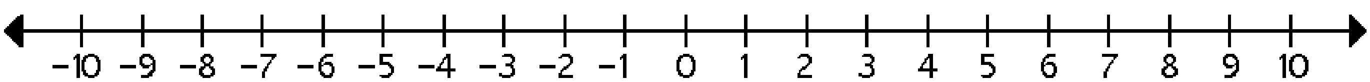
a. $x > 1$ and $x > 4$

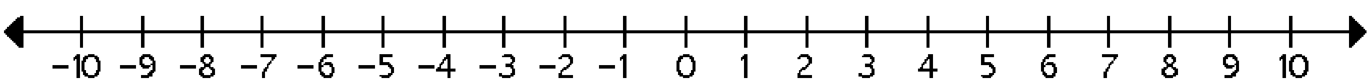


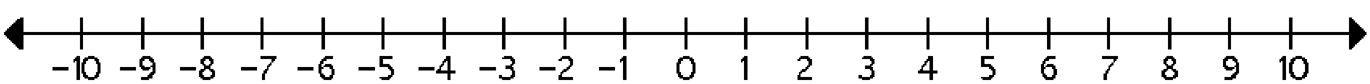




b. $x < 6$ and $x > -2$



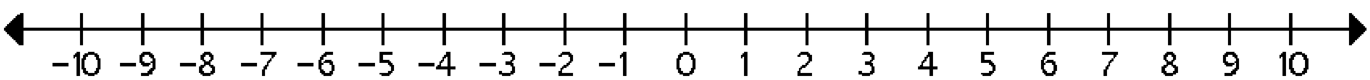




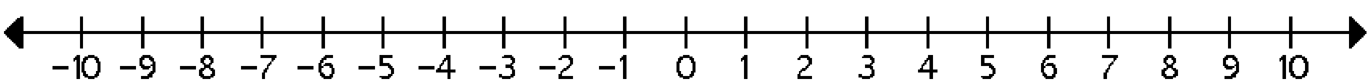
If ______________, the compound inequality ___________ and ____________ can be written in the shorter form ____________________________.

Example 3: Solve and graph the solution set:

a. $7 < x + 5 < 11$



b. $3 \le 4x - 3 < 19$



COMPOUND INEQUALITIES INVOLVING *OR*

If _____ and _____ are sets, we can form a new set consisting of all

____________________ that are in _____ or in _____ or in ___________

A and *B.* This is called the ____________________ of the two sets.

DEFINITION OF THE UNION OF SETS

The ______________________ of sets ____ and ____, written ____________, is the set of elements that are __________________ of set ______ or of set _____ or of ____________ sets. This definition can be expressed in set-builder notation as follows:

Example 4: Find the union of the sets.

a. $\{1,3,7\} \cup \{2,3,8\}$

b. $\{a,b,c\} \cup \{z\}$

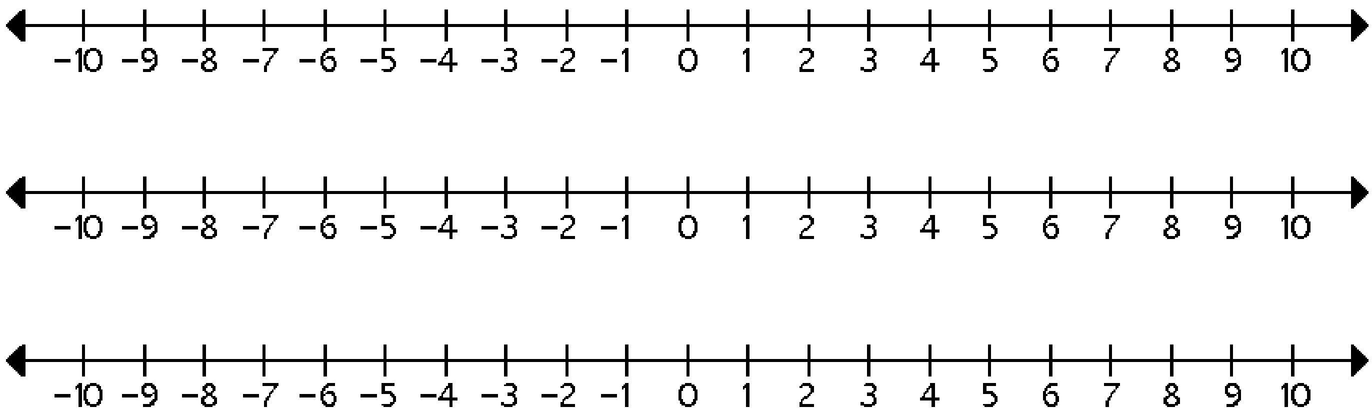
c. $\{-4,-3,-1\} \cup \{-2,3,4\}$

SOLVING COMPOUND INEQUALITIES INVOLVING *OR*

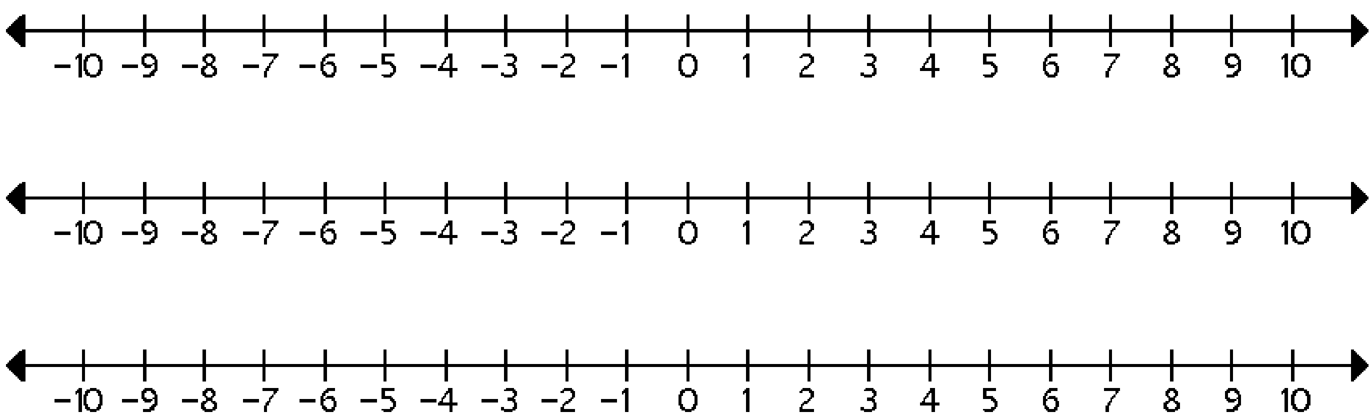
1. Solve each inequality ____________________.
2. Graph the solution set to ___________ inequality on a number line and take the _____________________ of these solution sets. This union appears as the portion of the number line representing the ______________ collection of numbers in the two graphs.

Example 5: Solve each compound inequality. Use graphs to show the solution set to each of the two given inequalities, as well as a third graph that shows the solution set of the compound inequality. Except for the empty set, express the solution set in interval notation.

a. $x > 0$ or $x \geq 4$



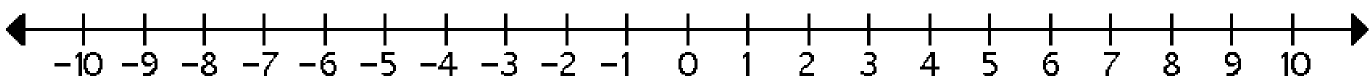
b. $x < -3$ or $x > 5$



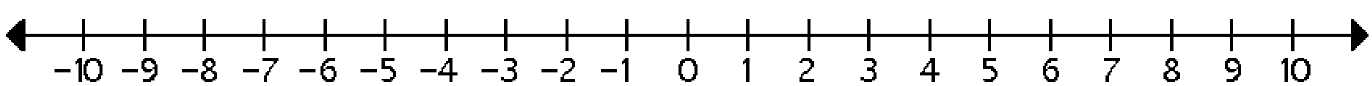
If _____________, the compound inequality ___________ and ____________ can be written in the shorter form ____________________________.

Example 6: Solve and graph the solution set:

a. $x-2(x+5)<12 \cup 5x+6>-1$



b. $4x-15>-10 \text{ or } \frac{x}{4}-1\le\frac{3}{4}$



Section 9.3: EQUATIONS AND INEQUALITIES INVOLVING ABSOLUTE VALUE

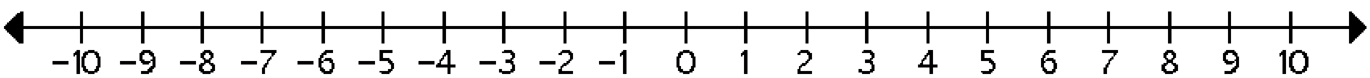
When you are done with your homework you should be able to…

- Solve absolute value equations
- Solve absolute value inequalities in the form $|u| < c$
- Solve absolute value inequalities in the form $|u| > c$
- Recognize absolute value inequalities with no solution or all real numbers as solutions
- Solve problems using absolute value inequalities

WARM-UP:

Graph the solutions of the inequality.

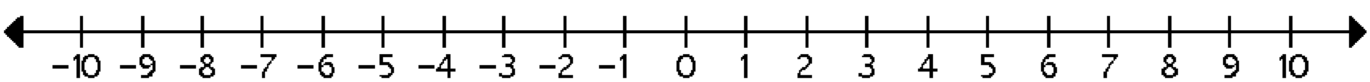
a. $-6 < x < 6$



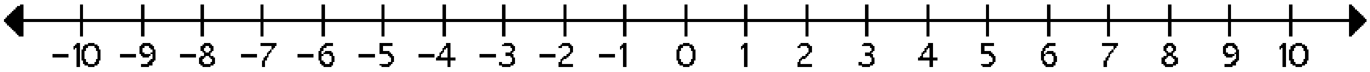
REWRITING AN ABSOLUTE VALUE EQUATION WITHOUT ABSOLUTE VALUE BARS

If ____ is a positive real number and ____ represents any _______________ expression, then _______________ is equivalent to ___________ or _________.

Consider $|x| = 6$.



Now consider $|x-3|=6$.



Example 1: Solve.

a. $|5x+7|=12$

b. $7|-x+11|=21$

c. $|x-4|-8=9$

d. $|x|+5=4$

REWRITING AN ABSOLUTE VALUE EQUATION WITH TWO ABSOLUTE VALUES WITHOUT ABSOLUTE VALUE BARS

If ______________, then ____________ or ______________.

Example 2: Solve.

$|2x-7|=|x-12|$

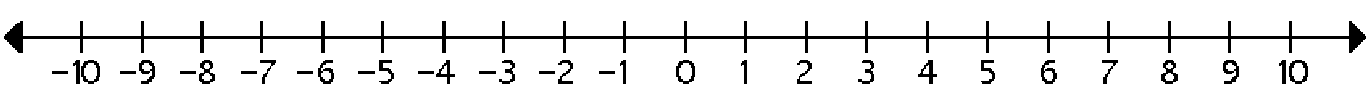
SOLVING ABSOLUTE VALUE INEQUALITIES OF THE FORM $|u|<c$

If ____ is a positive real number and ____ represents any ________________ expression, then

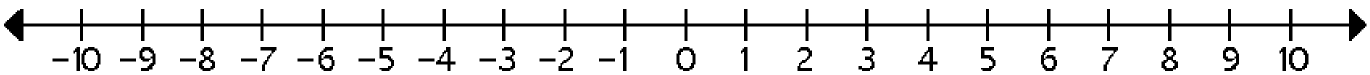
This rule is valid if __________ is replaced by __________.

Example 3: Solve and graph the solution set on a number line:

a. $|x|<6$



b. $-3|2x+7|+8 \geq -1$



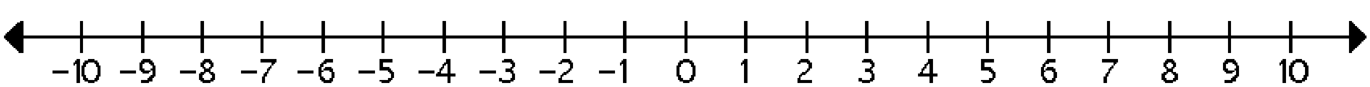
SOLVING ABSOLUTE VALUE INEQUALITIES OF THE FORM $|u| > c$

If _____ is a positive real number and _____ represents any ___________________ expression, then

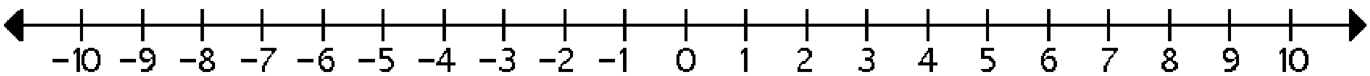
This rule is valid if ___________ is replaced by ___________.

Example 4: Solve and graph the solution set on a number line:

a. $|x| > 6$



b. $5|12x-1|-10 \geq 2$



ABSOLUTE VALUE INEQUALITIES WITH UNUSUAL SOLUTION SETS

If _____ is an algebraic expression and ______ is a __________________ number,

i. The inequality _________________ has ______ solution.

ii. The inequality _________________ is __________ for all real numbers for which ______ is defined.

APPLICATION

The inequality $|T-50| \leq 22$ describes the range of monthly average temperature T, in degrees Fahrenheit, for Albany, New York. Solve the inequality and interpret the solution.

Section 10.1: RADICAL EXPRESSIONS AND FUNCTIONS

When you are done with your homework you should be able to…

- π Evaluate square roots
- π Evaluate square root functions
- π Find the domain of square root functions
- π Use models that are square root functions
- π Simplify expressions of the form $\sqrt{a^2}$
- π Evaluate cube root functions
- π Simplify expressions of the form $\sqrt[3]{a^3}$
- π Find even and odd roots
- π Simplify expressions of the form $\sqrt[n]{a^n}$

WARM-UP:

1. Fill in the blank.

a. $5 \cdot \_\_ = 5^2$

b. $x^3 \cdot \_\_ = x^6$

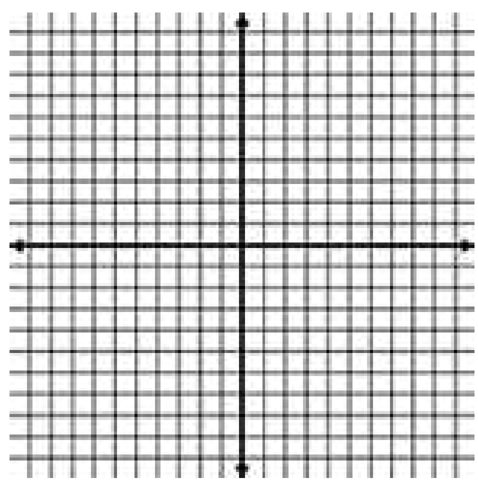
c. $\left(y^2\right)^{\_\_} = y^{16}$

d. $(-16)^2 = \_\_\_\_$

e. $-(16)^2 = \_\_\_\_\_$

2. Solve $|x| = 3$.

3. Graph $f(x) = \sqrt{x}$



DEFINITION OF THE PRINCIPAL SQUARE ROOT

If _____ is a nonnegative real number, the __________________ number ____ such that _________________, denoted by _________________, is the _________________ _____________ _______________ of _____.

Example 1: Evaluate.

a. $\sqrt{169}$

b. $\sqrt{0.04}$

c. $\sqrt{\frac{49}{64}}$

d. $\sqrt{36+64}$

e. $\sqrt{36}+\sqrt{64}$

SQUARE ROOT FUNCTIONS

Because each _____________________ number, ____, has precisely one principal square root, ________, there is a square root function defined by

The domain of this function is ____________. We can graph _____________ by selecting nonnegative real numbers for _____. It is easiest to pick perfect ______________.

How is this different than the graph we sketched in the warm-up?

Example 2: Find the indicated function value.

a. $f(x)=\sqrt{6x+10};\ f(1)$

b. $g(x)=-\sqrt{50-2x};\ f(5)$

Example 3: Find the domain of $f(x)=\sqrt{10x-7}$

SIMPLIFYING $\sqrt{a^2}$

For any real number *a*,

In words, the principal square root of ______ is the _____________ _________ of _____.

Example 4: Simplify each expression.

a. $\sqrt{(-9)^2}$

b. $\sqrt{(x-23)^2}$

c. $\sqrt{100x^{10}}$

d. $\sqrt{x^2-14x+49}$

DEFINITION OF THE CUBE ROOT OF A NUMBER

The cube root of a real number *a* is written ____________.

____________________ means that _________________.

CUBE ROOT FUNCTIONS

Unlike square roots, the cube root of a negative number is a ____________ number. All real numbers have cube roots. Because every ____________________ number, ____, has precisely one cube root, ________, there is a cube root function defined by

The domain of this function is ____________. We can graph _____________ by selecting real numbers for _____. It is easiest to pick perfect ____________.

SIMPLIFYING $\sqrt[3]{a^3}$

For any real number *a*,

In words, the cube root of any expression ___________ is that expression.

Example 5: Find the indicated function value.

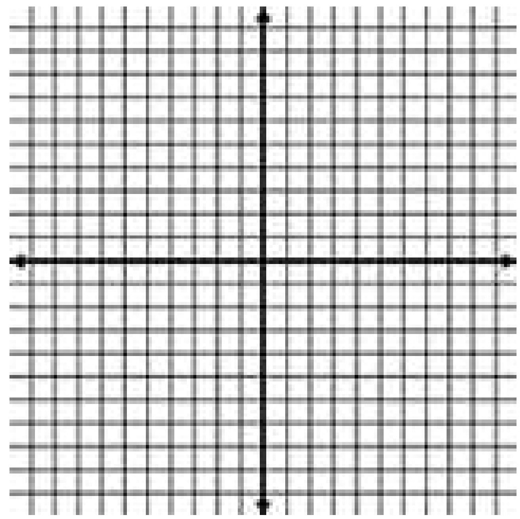
a. $f(x)=\sqrt[3]{x-20};\ f(12)$

b. $g(x)=\sqrt[3]{2x};\ g(32)$

Example 6: Graph the following functions by plotting points.

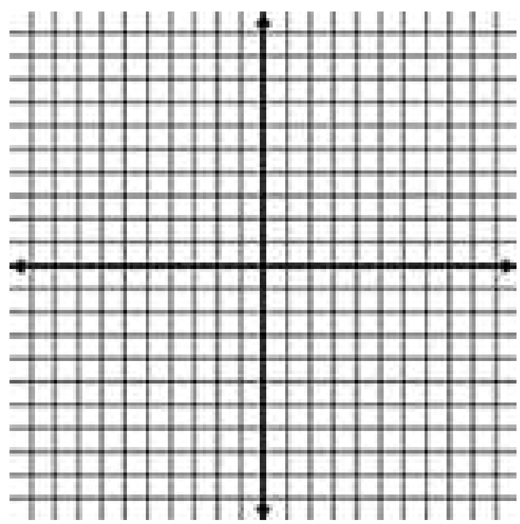
a. $f(x)=\sqrt{x+1}$

x	$f(x)=\sqrt{x+1}$	$(x, f(x))$



b. $g(x)=\sqrt[3]{x}$

x	$g(x)=\sqrt[3]{x}$	$(x, g(x))$



SIMPLIFYING $\sqrt[n]{a^n}$

For any real number *a*,

1. If *n* is even, _______________.

2. If *n* is odd, _______________.

Example 7: Simplify.

a. $\sqrt[6]{x^6}$

b. $\sqrt[5]{(2x-1)^5}$

c. $\sqrt[8]{(-2)^8}$

APPLICATION

Police use the function $f(x)=\sqrt{20x}$ to estimate the speed of a car, $f(x)$, in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road. A motorist is involved in an accident. A police officer measures the car's skid marks to be 45 feet long. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her?

Section 10.2: RATIONAL EXPONENTS

When you are done with your homework you should be able to…

- π Use the definition of $a^{\frac{1}{n}}$
- π Use the definition of $a^{\frac{m}{n}}$
- π Use the definition of $a^{-\frac{m}{n}}$
- π Simplify expressions with rational exponents
- π Simplify radical expressions using rational exponents

WARM-UP:

1. $\frac{1}{2}-\frac{3}{8}$

2. Simplify $\frac{x^2y^5}{\left(2x^3\right)^{-3}}$

THE DEFINITION OF $a^{\frac{1}{n}}$

If _____________ represents a real number and __________ is an integer, then

If *n* is even, *a* must be __________________. If *n* is odd, *a* can be any real number.

Example 1: Use radical notation to rewrite each expression. Simplify, if possible.

a. $400^{\frac{1}{2}}$

b. $\left(7xy^2\right)^{\frac{1}{3}}$

c. $\left(-32\right)^{\frac{1}{5}}$

Example 2: Rewrite with rational exponents.

a. $\sqrt[4]{12st}$

b. $\sqrt[3]{\frac{3z^2}{10}}$

c. $\sqrt{5xyz}$

THE DEFINITION OF $a^{\frac{m}{n}}$

If ____________ represents a real number, _______ is a positive rational number reduced to lowest terms, and _________ is an integer, then

and

Example 3: Use radical notation to rewrite each expression. Simplify, if possible.

a. $16^{\frac{3}{4}}$

b. $(-729)^{\frac{2}{3}}$

c. $(9)^{\frac{5}{2}}$

Example 4: Rewrite with rational exponents.

a. $\sqrt[3]{12^4}$

b. $\sqrt[5]{\left(\frac{x}{y}\right)^4}$

c. $\sqrt{(11t)^3}$

THE DEFINITION OF $a^{-\frac{m}{n}}$

If _____________ is a nonzero real number, then

Example 5: Rewrite each expression with a positive exponent. Simplify, if possible.

a. $144^{-\frac{1}{2}}$ b. $(-8)^{-\frac{2}{3}}$ c. $(32)^{-\frac{3}{5}}$

PROPERTIES OF RATIONAL EXPONENTS

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m b^n =$ ____________________.

2. $\frac{b^m}{b^n} =$ ____________________.

3. $(b^m)^n =$ ____________________.

4. $(ab)^n =$ ____________________.

5. $\left(\frac{a}{b}\right)^n =$ ____________________.

Example 6: Use properties of rational exponents to simplify each expression. Assume that all variables represent positive numbers.

a. $5^{\frac{2}{3}} \cdot 5^{\frac{1}{3}}$

b. $\left(125x^9y^6\right)^{\frac{1}{3}}$

c. $\dfrac{\left(2y^{\frac{1}{5}}\right)^4}{y^{\frac{3}{10}}}$

SIMPLIFYING RADICAL EXPRESSIONS USING RATIONAL EXPONENTS

1. Rewrite each radical expression as an ______________________ expression with a __________________ __________________.
2. Simplify using ________________ of rational exponents.
3. _______________ in radical notation if rational exponents still appear.

Example 7: Use rational exponents to simplify. If rational exponents appear after simplifying, write the answer in radical notation. Assume that all variables represent positive numbers.

a. $\left(\sqrt[3]{xy}\right)^{21}$

b. $\sqrt{3} \cdot \sqrt[3]{3}$

c. $\dfrac{\sqrt[4]{a^3 b^3}}{\sqrt{ab}}$

Section 10.3: MULTIPLYING AND SIMPLIFYING RADICAL EXPRESSIONS

When you are done with your homework you should be able to…

- π Use the product rule to multiply radicals
- π Use factoring and the product rule to simplify radicals
- π Multiply radicals and then simplify

WARM-UP:

1. Use properties of rational exponents to simplify each expression. Assume that all variables represent positive numbers.

a. $\frac{4^{\frac{2}{3}}}{4^{\frac{1}{3}}}$

b. $\left(196x^{10}y^{22}\right)^{\frac{1}{2}}$

2. Factor out the greatest common factor.

$8x^{\frac{1}{4}}+16x$

3. Multiply

$\left(x^{\frac{1}{2}}+3\right)\left(x^{\frac{3}{2}}-10\right)$

THE PRODUCT RULE FOR RADICALS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

The _________________ of two ___________ ______________ is the ________

root of the ________________ of the radicands.

Example 1: Multiply.

a. $\sqrt{2}\cdot\sqrt{11}$

b. $\sqrt[3]{4x}\cdot\sqrt[3]{12x}$

c. $\sqrt{x-1}\cdot\sqrt{x+1}$

SIMPLIFYING RADICAL EXPRESSIONS BY FACTORING

A radical expression whose index is *n* is __________________ when its radicand

has no ________________ that are perfect ________ powers. To simplify, use the following procedure:

1. Write the radicand as the ________________ of two factors, one of which is

   the _______________ perfect ___________ power.

2. Use the _______________ rule to take the _______ root of each factor.

3. Find the ________ root of the perfect *n*th power.

Example 2: Simplify by factoring. Assume that all variables represent positive numbers.

a. $\sqrt{12}$

b. $\sqrt[3]{81x^5}$

c. $\sqrt{288x^{11}y^{14}z^3}$

For the remainder of this chapter, in situations that do not involve functions, we will **assume that no radicands involve negative quantities raised to even powers. Based upon this assumption, absolute value bars are not necessary when taking even roots.

SIMPLIFYING WHEN VARIABLES TO EVEN POWERS IN A RADICAND ARE NONNEGATIVE QUANTITIES

For any ________________________ real number *a*,

Example 3: Simplify.

a. $\sqrt{108x^4y^3}$

b. $\sqrt[5]{64x^8y^{10}z^5}$

c. $\sqrt[4]{32x^{12}y^{15}}$

Example 4: Multiply and simplify.

a. $\sqrt{15xy} \cdot \sqrt{3xy}$

b. $\sqrt[3]{10x^2 y} \cdot \sqrt[3]{200x^2 y^2}$

Example 5: Simplify.

a. $\sqrt{5xy} \cdot \sqrt{10xy^2}$

b. $\sqrt[5]{8x^4 y^3 z^3} \cdot \sqrt[5]{8xy^9 z^8}$

c. $\left(2x^2 y \sqrt[4]{8xy}\right)\left(-32xy^2 \sqrt[4]{2x^2 y^3}\right)$

Section 10.4: ADDING, SUBTRACTING, AND DIVIDING RADICAL EXPRESSIONS

When you are done with your 10.4 homework you should be able to…

- π Add and subtract radical expressions
- π Use the quotient rule to simplify radical expressions
- π Use the quotient rule to divide radical expressions

WARM-UP:

Simplify.

a. $\dfrac{8x^3y^5}{2x^{-2}y^2}$

b. $3xy^2\sqrt[3]{16x^2y^2}$

THE QUOTIENT RULE FOR RADICALS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and __________, then

The ________ root of a ________________ is the ______________ of the

________ roots of the __________________ and __________________.

Example 1: Simplify using the quotient rule.

a. $\sqrt{\dfrac{20}{9}}$

b. $\sqrt[3]{\dfrac{x^6}{27y^{12}}}$

DIVIDING RADICAL EXPRESSIONS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and __________, then

To ____________ two radical expressions with the SAME ___________, divide the radicands and retain the ______________ _____________.

Example 2: Divide and, if possible, simplify.

a. $\dfrac{\sqrt{120x^4}}{\sqrt{3x}}$

b. $\dfrac{\sqrt[3]{128x^4y^2}}{\sqrt[3]{2xy^{-4}}}$

Example 3: Perform the indicated operations.

a. $\sqrt{2}+5\sqrt{2}$

c. $\dfrac{\sqrt{27}}{2}+\dfrac{\sqrt{75}}{7}$

b. $-\sqrt{20x^3}+3x\sqrt{80x}$

d. $\dfrac{16x^4\sqrt[3]{48x^3y^2}}{8x^3\sqrt[3]{3x^2y}}-\dfrac{20\sqrt[3]{2x^3y}}{4\sqrt[3]{x^{-1}}}$

10.5: MULTIPLYING RADICALS WITH MORE THAN ONE TERM AND RATIONALIZING DENOMINATORS

When you are done with your 10.5 homework you should be able to…

- π Multiply radical expressions with more than one term
- π Use polynomial special products to multiply radicals
- π Rationalize denominators containing one term
- π Rationalize denominators containing two terms
- π Rationalize numerators

WARM-UP:

Multiply.

a. $x^{\frac{1}{2}}(x-3)$

b. $(x^2-5)(x^2+5)$

c. $(3x-1)^2$

MULTIPLYING RADICAL EXPRESSIONS WITH MORE THAN ONE TERM

Radical expressions with more than one term are multiplied in much the same way

as ______________________ with more than one term are multiplied.

Example 1: Multiply.

a. $\sqrt{5}\left(x+\sqrt{10}\right)$

c. $\left(3\sqrt{3}-4\sqrt{2}\right)\left(6\sqrt{3}-10\sqrt{2}\right)$

b. $\sqrt[3]{y^2}\left(\sqrt[3]{16}-\sqrt[3]{y}\right)$

Example 2: Multiply.

a. $\left(x-\sqrt{10}\right)\left(x+\sqrt{10}\right)$

b. $\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$

c. $\left(\sqrt{3}+\sqrt{15}\right)^2$

CONJUGATES

Radical expressions that involve the ____________ and _________________ of the ____________ two terms are called _____________________.

RATIONALIZING DENOMINATORS CONTAINING ONE TERM occurs when you _______________ a radical expression as an ____________________ expression in which the denominator no longer contains any ________________.

When the denominator contains a _______________ radical with an *n*th root, multiply the ________________ and the ____________________ by a radical of index *n* that produces a perfect __________ power in the denominator's radicand.

Example 3: Rationalize each denominator.

a. $\frac{2}{\sqrt{3}}$

b. $\sqrt[3]{\frac{13}{2}}$

c. $\sqrt{\dfrac{5}{6xy}}$

d. $\dfrac{4x}{\sqrt[4]{8xy^3}}$

RATIONALIZING DENOMINATORS CONTAINING TWO TERMS

When the denominator contains two terms with one or more _____________ roots, **multiply the ________________ and the ____________________ by the ________________ of the denominator.**

Example 4: Rationalize each denominator.

a. $\dfrac{12}{1-\sqrt{3}}$

b. $\dfrac{6}{\sqrt{11}+\sqrt{5}}$

c. $\dfrac{2\sqrt{3}+7\sqrt{7}}{2\sqrt{3}-7\sqrt{7}}$

d. $\dfrac{\sqrt{x}+8}{\sqrt{x}+3}$

RATIONALIZING NUMERATORS

To rationalize a numerator, **multiply by___________ to eliminate the radical in the ____________________.**

Example 5: Rationalize each numerator.

a. $\sqrt{\dfrac{3}{2}}$

b. $\frac{\sqrt[3]{5x^2}}{4}$

c. $\frac{\sqrt{x}-\sqrt{2}}{x-2}$

Section 10.6: RADICAL EQUATIONS

When you are done with your homework you should be able to…

- π Solve radical equations
- π Use models that are radical functions to solve problems

WARM-UP:

Solve:

$$2x^2 - 3x = 5$$

SOLVING RADICAL EQUATIONS CONTAINING *n*th ROOTS

1. If necessary, arrange terms so that __________ radical is _______________ on one side of the equation.
2. Raise __________ sides of the equation to the ________ power to eliminate the *n*th root.
3. ___________the resulting equation. If this equation still contains radicals,

   ______________ steps 1 and 2!

4. _______________ all proposed solutions in the ________________ equation.

Example 1: Solve.

a. $\sqrt{5x-1}=8$

b. $\sqrt{2x+5}+11=6$

c. $x=\sqrt{6x+7}$

d. $\sqrt[3]{4x-3}-5=0$

e. $\sqrt{x+2}+\sqrt{3x+7}=1$

f. $2\sqrt{x-3}+4=x+1$

g. $2\left(x-1\right)^{\frac{1}{3}}=\left(x^2+2x\right)^{\frac{1}{3}}$

Example 2: If $f(x)=x-\sqrt{x-2}$, find all values of x for which $f(x)=4$.

Example 3: Solve $r=\sqrt{\frac{A}{4\pi}}$ for A .

Example 4: Without graphing, find the x-intercept of the function $f(x)=\sqrt{2x-3}-\sqrt{2x}+1$.

APPLICATION

A basketball player's hang time is the time spent in the air when shooting a basket. The formula $t=\frac{\sqrt{d}}{2}$ models hang time, t, in seconds, in terms of the vertical distance of a player's jump, d, in feet.

When Michael Wilson of the Harlem Globetrotters slam-dunked a basketball 12 feet, his hang time for the shot was approximately 1.16 seconds. What was the vertical distance of his jump, rounded to the nearest tenth of a foot?

Section 10.7: COMPLEX NUMBERS

When you are done with your homework you should be able to…

- Express square roots of negative numbers in terms of *i*
- Add and subtract complex numbers
- Multiply complex numbers
- Divide complex numbers
- Simplify powers of *i*

WARM-UP:

Rationalize the denominator:

a. $\frac{5}{\sqrt{x}}$

b. $\frac{3-\sqrt{x}}{3+\sqrt{x}}$

THE IMAGINARY UNIT *i*

The imaginary unit ____ is defined as

THE SQUARE ROOT OF A NEGATIVE NUMBER

If *b* is a positive real number, then

Example 1: Write as a multiple of *i.*

a. $\sqrt{-100}$

b. $\sqrt{-50}$

COMPLEX NUMBERS AND IMAGINARY NUMBERS

The set of all numbers in the form

with real numbers *a* and *b,* and *i*, the imaginary unit, is called the set of

______________ ________________. The real number ___ is called the real

part and the real number ___ is called the imaginary part of the complex

number_____________. If ___________, then the complex number is called an

__________________ number.

Example 2: Express each number in terms of *i* and simplify, if possible.

a. $7+\sqrt{-4}$

b. $-3-\sqrt{-27}$

ADDING AND SUBTRACTING COMPLEX NUMBERS

1. $(a+bi)+(c+di)$ = ___________________________

2. $(a+bi)-(c+di)$ = ___________________________

Example 3: Add or subtract as indicated. Write the result in the form $a+bi$.

a. $(6+5i)+(4+3i)$

b. $(-7+3i)-(9-10i)$

MULTIPLYING COMPLEX NUMBERS

Multiplication of complex numbers is performed the same way as multiplication of ______________________, using the ______________________ property and the FOIL method. After completing the multiplication, we replace any occurrences of ______ with _____.

Example 4: Multiply.

a. $(5+8i)(4i-3)$

b. $(2+7i)(2-7i)$

c. $\left(3+\sqrt{-16}\right)^2$

CONJUGATES AND DIVISION

The _________________ of the complex number $a+bi$ is ____________. The _________________ of the complex number $a-bi$ is ____________. Conjugates are used to _______________ complex numbers. The goal of the division procedure is to obtain a real number in the _______________________. This real number becomes the denominator of _____ and _____ in ________________. By multiplying the numerator and denominator of the quotient by the ____________________ of the denominator, you will obtain this real number in the denominator.

Example 5: Divide and simplify to the form $a+bi$.

a. $\frac{9}{-8i}$

b. $\frac{3}{4+i}$

c. $\frac{5i}{2-3i}$

d. $\frac{6-3i}{4+2i}$

e. $\frac{1-i}{1+i}$

SIMPLIFYING POWERS OF *i*

1. Express the given power of *i* in terms of _____.

2. Replace _______ with ________ and simplify.

Example 6: Simplify.

a. i^{14}

b. i^{15}

c. i^{46}

d. $(-i)^6$

Section 11.1: THE SQUARE ROOT PROPERTY AND COMPLETING THE SQUARE; DISTANCE AND MIDPOINT FORMULAS

When you are done with your homework you should be able to…

- π Solve quadratic equations using the square root property
- π Complete the square of a binomial
- π Solve quadratic equations by completing the square
- π Solve problems using the square root property
- π Find the distance between two points
- π Find the midpoint of a line segment

WARM-UP:

Solve.

a. $(x-1)^2=4$

b. $(x-5)^2=0$

THE SQUARE ROOT PROPERTY

If u is an algebraic expression and d is a nonzero real number, then

if ______________, then _____________ or _______________.

Equivalently,

if ______________, then ______________.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^2 = 9$

b. $2x^2 - 10 = 0$

c. $4x^2 + 49 = 0$

d. $x^2 - 10x + 25 = 1$

e. $3(x+2)^2 = 36$

COMPLETING THE SQUARE

If $x^2 + bx$ is a binomial, then by adding $\left(\frac{b}{2}\right)^2$, which is the square of _______ the _________________ of _____, a perfect square trinomial will result.

$$x^2 + bx \text{ \_\_\_\_\_\_\_\_\_\_ } = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Example 2: Find $\left(\frac{b}{2}\right)^2$ for each expression.

a. $x^2 + 2x$

b. $x^2 - 12x$

c. $x^2 + 5x$

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Consider a quadratic equation in the form $ax^2 + bx + c$.

1. If $a \neq 1$, divide both sides of the equation by ______.
2. Isolate $x^2 + bx$.
3. Add _________ to BOTH sides of the equation.
4. Factor and simplify.
5. Apply the square root property.
6. Solve.
7. Check your solution(s) in the ____________________ equation.

Example 3: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^2+8x-2=0$

b. $x^2-3x-5=0$

c. $3x^2 - 6x = -2$

d. $4x^2 - 2x + 5 = 0$

A FORMULA FOR COMPOUND INTEREST

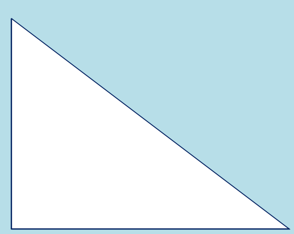
Suppose that an amount of money, ____, is invested at interest rate, ____, compounded annually. In ____ years, the amount, ____, or balance, in the account is given by the formula

Example 4: You invested $4000 in an account whose interest is compounded annually. After 2 years, the amount, or balance, in the account is $4300. Find the annual interest rate. Round to the nearest hundredth of a percent.

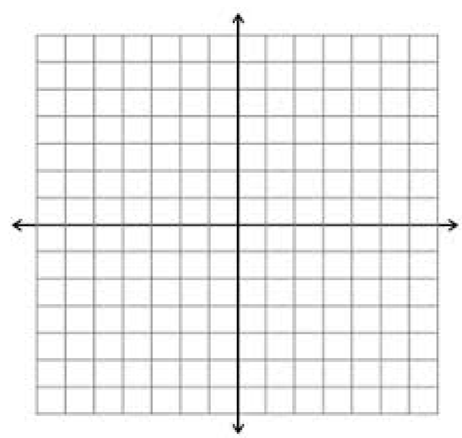
THE PYTHAGOREAN THEOREM

The sum of the squares of the _______________ of the ______________ of a ______________ triangle equals the ______________ of the _____________ of the ____________________.

If the legs have lengths _____ and _____, and the hypotenuse has length ____, then



Example 5: The doorway into a room is 4 feet wide and 8 feet high. What is the diameter of the largest circular tabletop that can be taken through this doorway diagonally?



THE DISTANCE FORMULA

The distance, ____, between the points ___________ and _________ in the rectangular coordinate system is

Example 6: Find the distance between each pair of points.

a. $(5,1)$ and $(8,-2)$

b. $\left(2\sqrt{3},\sqrt{6}\right)$ and $\left(-\sqrt{3},5\sqrt{6}\right)$

THE MIDPOINT FORMULA

Consider a line segment whose endpoints are _____________ and ____________.

The coordinates of the segment's midpoints are

Example 7: Find the midpoint of the line segment with the given endpoints.

a. $(10,4)$ and $(2,6)$

b. $\left(-\frac{2}{5},\frac{7}{15}\right)$ and $\left(-\frac{2}{5},-\frac{4}{15}\right)$

Section 11.2: THE QUADRATIC FORMULA

When you are done with your homework you should be able to…

- π Solve quadratic equations using the quadratic formula
- π Use the discriminant to determine the number and type of solutions
- π Determine the most efficient method to use when solving a quadratic equation
- π Write quadratic equations from solutions
- π Use the quadratic formula to solve problems

WARM-UP:

Solve for *x* by completing the square and applying the square root property.

$$ax^2 + bx + c = 0$$

THE QUADRATIC FORMULA

The solutions of a quadratic equation in standard form $ax^2+bx+c=0$, with $a \neq 0$, are given by the **quadratic formula**:

STEPS FOR USING THE QUADRATIC FORMULA

1. Write the quadratic equation in ___________ form (__________________).

2. Determine the numerical values for ____, _____, and _____.

3. Substitute the values of _____, _____, and ____ into the quadratic formula and ________________ the expression.

4. Check your solution(s) in the ________________ equation.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $4x^2+3x=2$

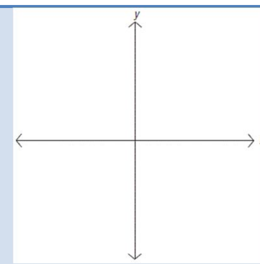
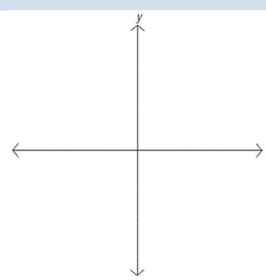
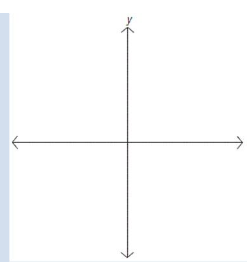
b. $3x^2 = 4x - 6$

c. $2x(x+4) = 3x - 3$

d. $x^2 + 5x - 10 = 0$

THE DISCRIMINANT

The quantity _________________, which appears under the _______________ sign in the _______________ formula, is called the ____________________. The discriminant determines the ________________ and __________ of solutions of quadratic equations.

DISCRIMINANT $b^2 - 4ac$	KINDS OF SOLUTIONS TO $ax^2 + bx + c = 0$	GRAPH OF $y = ax^2 + bx + c$
$b^2 - 4ac > 0$		 2 _______________
$b^2 - 4ac = 0$		 1 ________________
$b^2 - 4ac < 0$		 NO _______________

Example 2: Compute the discriminant. Then determine the number and type of solutions.

a. $2x^2 - 4x + 3 = 0$

b. $4x^2 = 20x - 25$

c. $x^2 + 2x - 3 = 0$

DESCRIPTION AND FORM OF THE QUADRATIC EQUATION	MOST EFFICIENT SOLUTION METHOD
$ax^2 + bx + c = 0$, and $ax^2 + bx + c$ can be easily factored.	____________ and use the _______ ____________ principle.
$ax^2 + c = 0$ The quadratic equation has no ______ term (_______).	Isolate ______ and use the ___________ ____________ property.
$u^2 = d$; u is a first-degree polynomial.	Use the _____________ _________ property.
$ax^2 + bx + c = 0$, and $ax^2 + bx + c$ cannot factored or the factoring is too difficult.	Use the ________________ formula.

THE ZERO-PRODUCT PRINCIPLE IN REVERSE

If _____ or _______, then _____________.

Example 3: Write a quadratic equation with the given solution set.

a. $\{-2, 6\}$

b. $\{-\sqrt{3}, \sqrt{3}\}$

c. $\{2+i, 2-i\}$

Example 4: The hypotenuse of a right triangle is 6 feet long. One leg is 2 feet shorter than the other. Find the lengths of the legs.

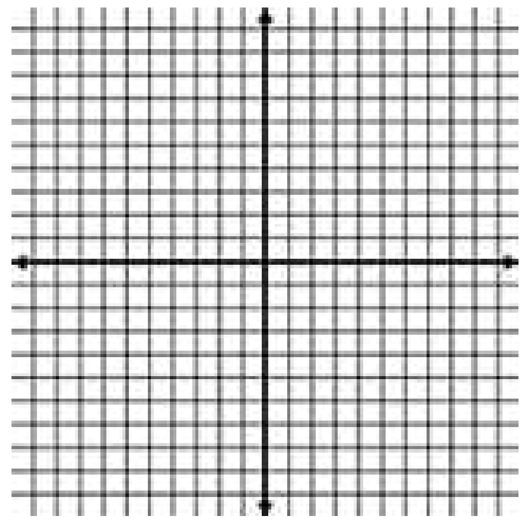
When you are done with your homework you should be able to…

- π Recognize characteristics of parabolas
- π Graph parabolas in the form $f(x)=a(x-h)^2+k$
- π Graph parabolas in the form $f(x)=ax^2+bx+c$
- π Determine a quadratic function's minimum or maximum value
- π Solve problems involving a quadratic function's minimum or maximum value

WARM-UP: Graph the following functions by plotting points.

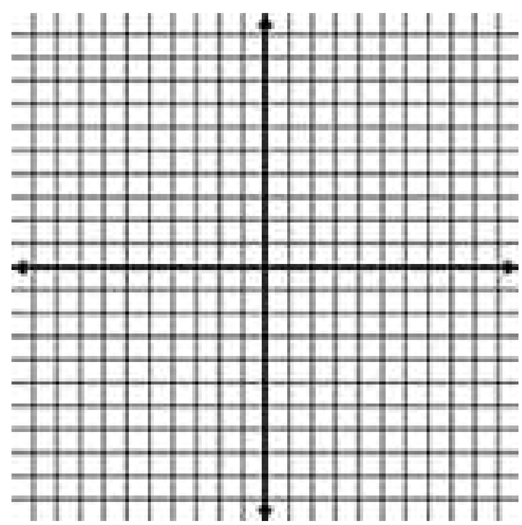
a. $f(x)=x^2$

x	$f(x)=x^2$	$(x, f(x))$



b. $f(x)=-x^2$

x	$f(x)=-x^2$	$(x, f(x))$



QUADRATIC FUNCTIONS IN THE FORM $f(x)=a(x-h)^2+k$

The graph of

is a ___________________ whose _____________ is the point ____________.

The parabola is ___________________ with respect to the line __________. If

___________, the parabola opens upwards; if ____________, the parabola opens

_____________________.

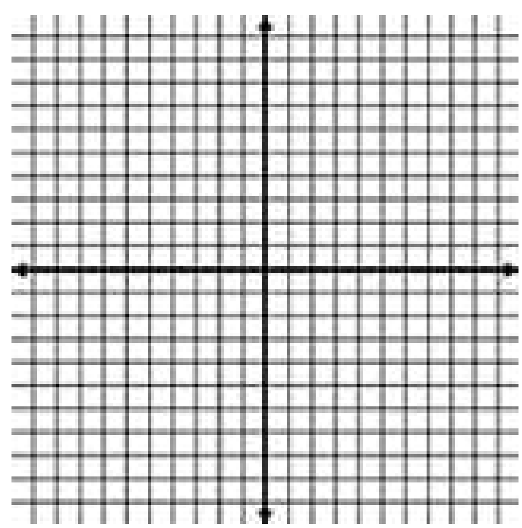
$$f(x)=a(x-h)^2+k$$

GRAPHING QUADRATIC FUNCTIONS WITH EQUATIONS IN THE FORM $f(x)=a(x-h)^2+k$

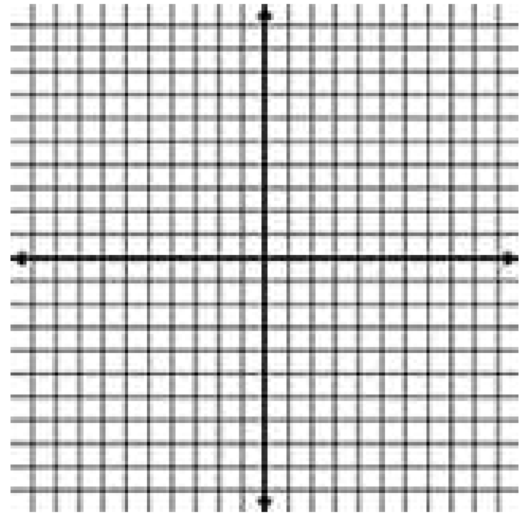
1. Determine whether the ________________ opens _______________ or ____________________. If ___________ the parabola opens upward and if ________________ the parabola opens ______________________.
2. Determine the ________________ of the parabola. The vertex is _________.
3. Find any _________________ by solving _________________.
4. Find the ______________ by computing _______.
5. Plot the _________________, the ______________, and additional points as necessary. Connect these points with a _______________ curve that is shaped like a _____________ or an inverted bowl.

Example 1: Use the vertex and intercepts to sketch the graph of each quadratic function. Use the graph to identify the function's range.

a. $f(x)=(x-1)^2-2$



b. $f(x) = 2(x+2)^2 - 1$



THE VERTEX OF A PARABOLA WHOSE EQUATION IS $f(x) = ax^2 + bx + c$

The parabola's vertex is ________________________. The ___________________ is

________________ and the ______________________ is found by substituting the

_____________________ into the parabola's equation and _____________________

the function at this value of _________.

Example 2: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

a. $f(x) = 3x^2 - 12x + 1$

b. $f(x) = -2x^2 + 7x - 4$

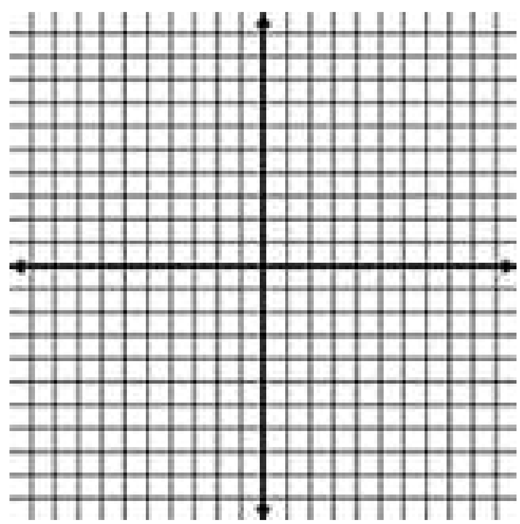
c. $f(x) = -3(x-2)^2 + 12$

GRAPHING QUADRATIC FUNCTIONS WITH EQUATIONS IN THE FORM $f(x)=ax^2+bx+c$

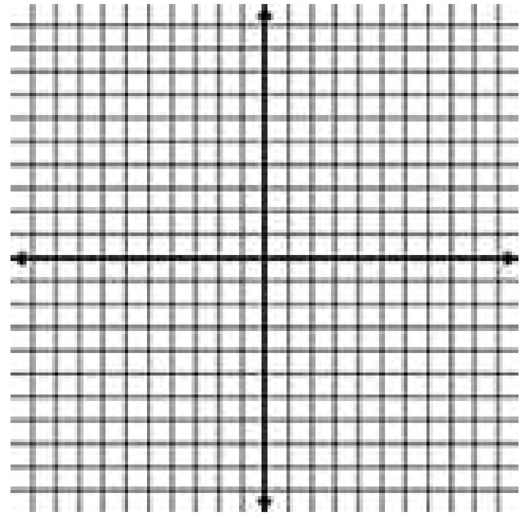
1. Determine whether the _______________ opens ______________ or __________________. If __________ the parabola opens upward and if _______________ the parabola opens _____________________.
2. Determine the __________ of the parabola. The vertex is _____________.
3. Find any ________________ by solving ________________.
4. Find the _____________ by computing ______.
5. Plot the ________________, the _____________, and additional points as necessary. Connect these points with a ______________ curve that is shaped like a ____________ or an inverted bowl.

Example 3: Use the vertex and intercepts to sketch the graph of each quadratic function. Use the graph to identify the function's range.

a. $f(x)=x^2-2x-15$



b. $f(x) = 5 - 4x - x^2$



MINIMUM AND MAXIMUM: QUADRATIC FUNCTIONS

Consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If _______, then ______ has a __________________ that occurs at _______.

   This _________________ ___________ is _______________.

2. If _______, then ______ has a __________________ that occurs at _______.

   This _________________ ___________ is _______________.

In each case, the value of _____ gives the __________________ of the minimum or maximum value. The value of _____, or ___________, gives that minimum or maximum value.

Example 4: Among all pairs of numbers whose sum is 20, find a pair whose product is as large as possible. What is the maximum product?

Example 5: You have 200 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

Section 11.4: EQUATIONS QUADRATIC IN FORM

When you are done with your homework you should be able to…

- π Solve equations that are quadratic in form

WARM-UP: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $-5x^2+x=3$

b. $x^2=x-6$

EQUATIONS WHICH ARE QUADRATIC IN FORM

An equation that is __________________ in ____________ is one that can be

expressed as a quadratic equation using an appropriate ____________________.

In an equation that is quadratic in form, the ____________________ factor in one

term is the _________________ of the variable factor in the other variable

term. The third term is a __________________. By letting ____ equal the

variable factor that reappears squared, a quadratic equation in ____ will result.

Solve this quadratic equation for ____ using the methods you learned earlier.

Then use your substitution to find the values for the ________________ in the

__________________ equation.

Example 1: Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

a. $x^4 - 13x^2 + 36 = 0$

b. $x^4 + 4x^2 = 5$

c. $x + \sqrt{x} - 6 = 0$

d. $\left(x+3\right)^2+7\left(x+3\right)-18=0$

e. $x^{-2}-6x^{-1}=-4$

Section 12.1: EXPONENTIAL FUNCTIONS

When you are done with your homework you should be able to…

- π Evaluate exponential functions
- π Graph exponential functions
- π Evaluate functions with base *e*
- π Use compound interest formulas

WARM-UP:

Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

$$\left(x^2-2\right)^2-\left(x^2-2\right)=6$$

DEFINITION OF AN EXPONENTIAL FUNCTION

The **exponential function ____ with base ____** is defined by

where ____ is a _____________ constant other than ___ (____ and ____) and

___ is any real number.

Example 1: Determine if the given function is an exponential function.

a. $f(x) = 3^x$

b. $g(x) = (-4)^{x+1}$

Example 2: Evaluate the exponential function at $x = -2,\ 0,\ \text{and } 2$.

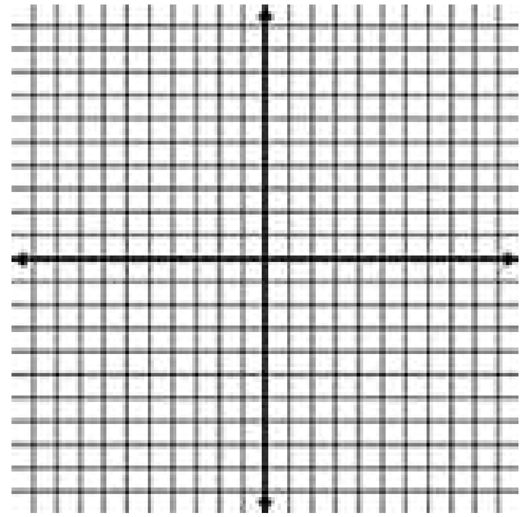
a. $f(x) = 2^x$

b. $g(x) = \left(\frac{1}{3}\right)^x$

Example 3: Sketch the graph of each exponential function.

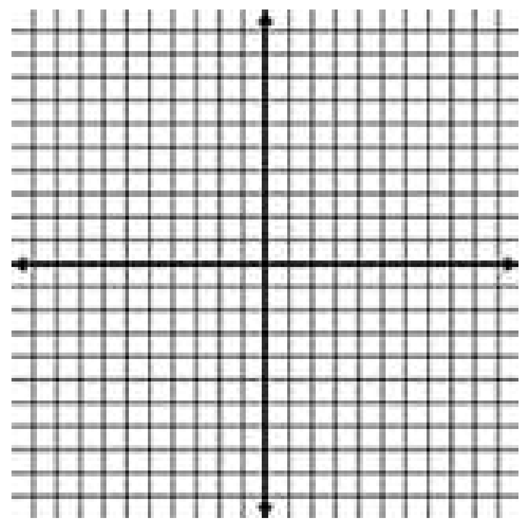
a. $f(x)=3^x$

x	$f(x)=3^x$	$(x, f(x))$



b. $g(x)=3^{-x}$

x	$g(x)=3^{-x}$	$(x, g(x))$

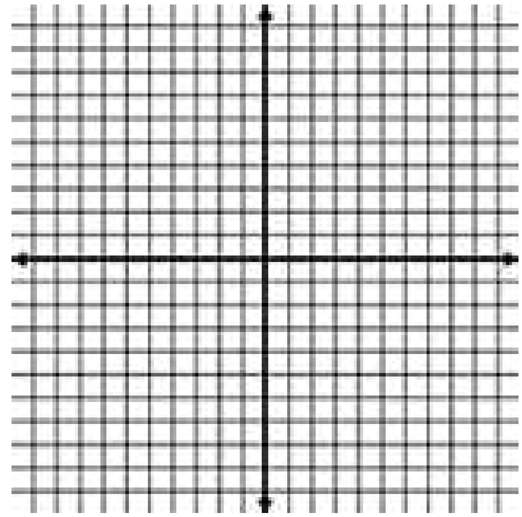


How are these two graphs related?

Example 4: Sketch the graph of each exponential function.

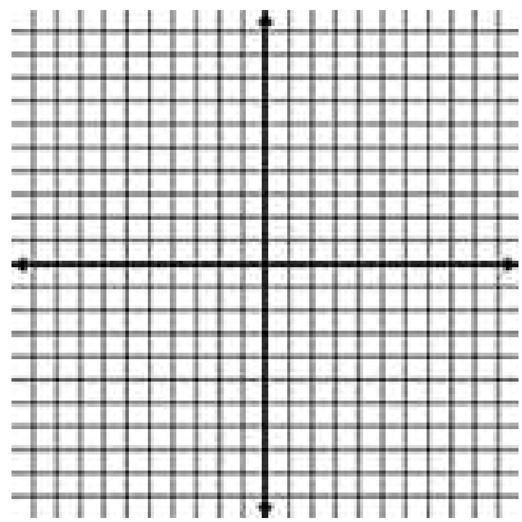
a. $f(x)=2^x$

x	$f(x)=2^x$	$(x, f(x))$



b. $g(x)=2^{x+1}$

x	$g(x)=2^{x+1}$	$(x, g(x))$



How are these two graphs related?

CHARACTERISTICS OF EXPONENTIAL FUNCTIONS OF THE FORM $f(x)=b^x$

1. The domain of $f(x)=b^x$ consists of all real numbers: _________. The range of $f(x)=b^x$ consists of all ______________ real numbers: _________.
2. The graphs of all exponential functions of the form $f(x)=b^x$ pass through the point ________ because ____________ (______). The _________________ is ___.
3. If _________, $f(x)=b^x$ has a graph that goes _____ to the ___________ and is an ______________ function. The greater the value of _____, the steeper the _______________.
4. If ________, $f(x)=b^x$ has a graph that goes __________ to the ________ and is a _________________ function. The smaller the value of ___, the steeper the _______________.
5. The graph of $f(x)=b^x$ approaches, but does not touch, the _____________. The line _______ is a ____________________ asymptote.

n	$\left(1+\frac{1}{n}\right)^n$
1	
2	
5	
10	
100	
1000	
10000	
100000	
1000000	
1000000000	

The irrational number ____, approximately __________, is called the ___________ base. The function _______________ is called the _______________ exponential function.

FORMULAS FOR COMPOUND INTEREST

After _____ years, the balance ____, in an account with principal ____ and annual interest rate ____ (in decimal form) is given by the following formulas:

1. For ___ compounding interest periods per year:

2. For continuous compounding:

Example 5: Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is

a. compounded semiannually:

b. compounded monthly:

c. compounded continuously:

Section 12.2: LOGARITHMIC FUNCTIONS

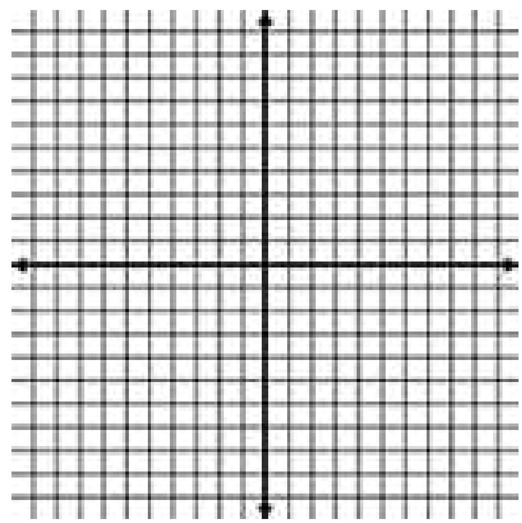
When you are done with your homework you should be able to…

- π Change from logarithmic to exponential form
- π Change from exponential to logarithmic form
- π Evaluate logarithms
- π Use basic logarithm properties
- π Graph logarithmic functions
- π Find the domain of a logarithmic function
- π Use common logarithms
- π Use natural logarithms

WARM-UP:

Graph $y = 2^x$.

x	$y = 2^x$	(x, y)



DEFINITION OF THE LOGARITHMIC FUNCTION

For _________ and ____________, __________,

__________________ is equivalent to __________________.

The function ______________ is the **logarithmic function with base ____**.

Example 1: Write each equation in its equivalent exponential form:

a. $\log_4 x = 2$

b. $y = \log_3 81$

Example 2: Write each equation in its equivalent logarithmic form:

a. $e^y = 9$

b. $b^4 = 16$

Example 3: Evaluate.

a. $\log_5 25$

b. $\log_{81} 9$

BASIC LOGARITHMIC PROPERTIES INVOLVING 1

1. $\log_b b =$ __________ "the power to which I raise _____to get _____ is ______"

2. $\log_b 1 =$ __________ "the power to which I raise _____to get _____ is ______"

INVERSE PROPERTIES OF LOGARITHMS

For __________ and __________,

1. $\log_b b^x =$ __________

2. $b^{\log_b x} =$ __________

Example 4: Evaluate.

a. $\log_6 6$

b. $\log_{12} 12^4$

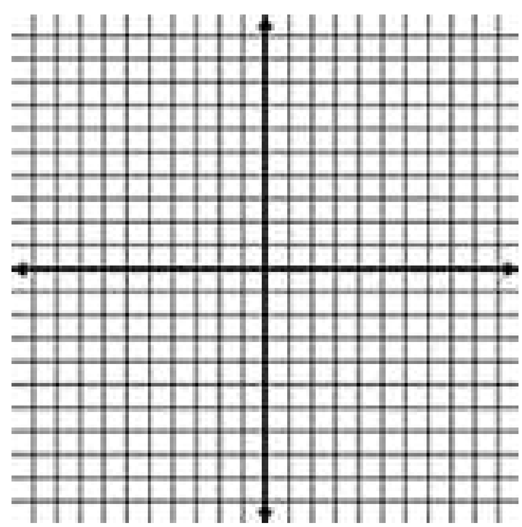
c. $\log_9 1$

d. $7^{\log_7 24}$

Example 5: Sketch the graph of each logarithmic function.

$f(x) = \log_3 x$

x	$f(x) = \log_3 x$	$(x, f(x))$



CHARACTERISTICS OF LOGARITHMIC FUNCTIONS OF THE FORM $f(x)=\log_b x$

1. The domain of $f(x)=\log_b x$ consists of all positive real numbers: _________.

   The range of $f(x)=\log_b x$ consists of all real numbers: _________.

2. The graphs of all logarithmic functions of the form $f(x)=\log_b x$ pass

   through the point ________ because ____________ . The

   __________________ is ___. There is no ___________________.

3. If _________, $f(x)=\log_b x$ has a graph that goes ____ to the __________

   and is an ______________ function.

4. If ________, $f(x)=\log_b x$ has a graph that goes _________ to the ________

   and is a _________________ function.

5. The graph of $f(x)=\log_b x$ approaches, but does not touch, the

   _____________. The line _______ is a ____________________ asymptote.

Example 6: Find the domain.

a. $f(x)=\log_2(x-4)$

b. $f(x)=\log_5(1-x)$

COMMON LOGARITHMS

The logarithmic function with base ______ is called the **common logarithmic function.** The function ___________________ is usually expressed as ___________________. A calculator with a LOG key can be used to evaluate common logarithms.

Example 7: Evaluate.

a. $\log 1000$

b. $\log 0.01$

PROPERTIES OF COMMON LOGARITHMS

1. $\log 1 =$ ________
2. $\log 10 =$ ________
3. $\log 10^{x} =$ ______
4. $10^{\log x} =$ ______

Example 8: Evaluate.

a. $\log 10^{3}$

b. $10^{\log 7}$

NATURAL LOGARITHMS

The logarithmic function with base ______ is called the **natural logarithmic function.** The function __________________ is usually expressed as __________________. A calculator with a LN key can be used to evaluate common logarithms.

PROPERTIES OF NATURAL LOGARITHMS

1. $\ln 1 =$ ________
2. $\ln e =$ _______
3. $\ln e^x =$ ______
4. $e^{\ln x} =$ ______

Example 9: Evaluate.

a. $\ln \frac{1}{e^6}$

b. $e^{\ln 300}$

Example 10: Find the domain of $f(x) = \ln(x-4)^2$.

Section 12.3: PROPERTIES OF LOGARITHMS

When you are done with your 12.3 homework you should be able to…

- π Use the product rule
- π Use the quotient rule
- π Use the power rule
- π Expand logarithmic expressions
- π Condense logarithmic expressions
- π Use the change-of-base property

WARM-UP:

Simplify.

a. $5^x \cdot 5^x$

b. $\frac{2^{3x}}{2^x}$

THE PRODUCT RULE

Let ___, ___, and ___ be positive real numbers with _________.

The logarithm of a product is the ________ of the _________________.

Example 1: Expand each logarithmic expression.

a. $\log_6(6x)$

b. $\ln(x \cdot x)$

THE QUOTIENT RULE

Let ___, ___, and ___ be positive real numbers with _________.

The logarithm of a quotient is the _____________ of the ________________.

Example 2: Expand each logarithmic expression.

a. $\log \frac{1}{x}$

b. $\log_4 \frac{x}{2}$

THE POWER RULE

Let ___ and ___ be positive real numbers with _________, and let ___ be any real number.

The logarithm of a number with an _______________ is the ______________ of the exponent and the _________________ of that number.

Example 3: Expand each logarithmic expression.

a. $\log x^2$

b. $\log_5 \sqrt{x}$

PROPERTIES FOR EXPANDING LOGARITHMIC EXPRESSIONS

For______ and _______:

1. _________________ $= \log_b M + \log_b N$

2. _________________ $= \log_b M - \log_b N$

3. _________________ $= p \log_b M$

Example 4: Expand each logarithmic expression.

a. $\log x^3 \sqrt[3]{y}$

b. $\log_4 \sqrt{\dfrac{x}{12y^5}}$

PROPERTIES FOR CONDENSING LOGARITHMIC EXPRESSIONS

For______ and _______:

1. _____________________ $= \log_b (MN)$

2. _____________________ $= \log_b \dfrac{M}{N}$

3. _____________________ $= \log_b M^p$

Example 5: Write as a single logarithm.

a. $3\ln x - \frac{1}{4}\ln(x-2)$

b. $\log_4 5 + 12\log_4(x+y)$

THE CHANGE-OF-BASE PROPERTY

For any logarithmic bases ___ and ___, and any positive number ___,

The logarithm of ____ with base ____ is equal to the logarithm of ___ with any new base divided by the logarithm of ___ with that new base.

Why would we use this property?

Example 6: Use common logarithms to evaluate $\log_5 23$.

Example 7: Use natural logarithms to evaluate $\log_5 23$.

What did you find out???

Section 12.4: EXPONENTIAL AND LOGARITHMIC EQUATIONS

When you are done with your 12.4 homework you should be able to…

- π Use like bases to solve exponential equations
- π Use logarithms to solve exponential equations
- π Use exponential form to solve logarithmic equations
- π Use the one-to-one property of logarithms to solve logarithmic equations
- π Solve applied problems involving exponential and logarithmic equations

WARM-UP:

Solve.

$$\frac{x-1}{5}=\frac{2}{5}$$

SOLVING EXPONENTIAL EQUATIONS BY EXPRESSING EACH SIDE AS A POWER OF THE SAME BASE

If ______________________, then ___________________.

1. Rewrite the equation in the form ___________________.
2. Set _______________.
3. Solve for the variable.

Example 1: Solve.

a. $10^{x^2-1}=100$

b. $4^{x+1}=8^{3x}$

USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

1. Isolate the ________________ expression.
2. Take the _________________ logarithm on both sides for base _____. Take the ________________ logarithm on both sides for bases other than 10.
3. Simplify using one of the following properties:
4. Solve for the variable.

Example 2: Solve.

a. $e^{2x}-6=32$

b. $\dfrac{3^{x-1}}{2}=5$

c. $10^{x}=120$

USING EXPONENTIAL FORM TO SOLVE LOGARITHMIC EQUATIONS
1. Express the equation in the form __________________. 2. Use the definition of a logarithm to rewrite the equation in exponential form: 3. Solve for the variable. 4. Check proposed solutions in the _______________ equation. Include in the solution set only values for which _________.

Example 3: Solve.

a. $\log_3 x - \log_3 (x-2) = 4$

b. $\log x + \log(x+21) = 2$

USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form ___________________. This form involves a _______________ logarithm whose coefficient is ___ on each side of the equation.

2. Use the one-to-one property to rewrite the equation without logarithms:

3. Solve for the variable.
4. Check proposed solutions in the _______________ equation. Include in the

   solution set only values for which __________ and ____________.

Example 4: Solve.

a. $2\log_6 x - \log_6 64 = 0$

b. $\log(5x+1) = \log(2x+3) + \log 2$

Section 12.5: EXPONENTIAL GROWTH AND DECAY; MODELING DATA

When you are done with your 12.5 homework you should be able to…

- Model exponential growth and decay

WARM-UP: Solve. Express the solution set in terms of logarithms. Then use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

a. $1250e^{0.065x} = 6250$

b. $4e^{7x} = 10273$

One of algebra's many applications is to _______________ the behavior of variables. This can be done with exponential ____________________ and _______________ models. With exponential growth or decay, quantities grow or decay ate a rate directly ______________________ to their size.

EXPONENTIAL GROWTH AND DECAY MODELS

The mathematical model for **exponential growth** or **decay** is given by

- If ___________, the function models the amount, or size, of a _______________ entity. ________ is the ________________ amount, or size, of the growing entity at time _____________, ________ is the amount at time ____, and _____ is a constant representing the ________________ rate.
- If ___________, the function models the amount, or size, of a _______________ entity. ________ is the ________________ amount, or size, of the decaying entity at time _____________, ________ is the amount at time ____, and _____ is a constant representing the ________________ rate.

Example 1: In 2000, the population of the Palestinians in the West Bank, Gaza Strip, and East Jerusalem was approximately 3.2 million, and by 2050 it is projected to grow to 12 million.

a. Use the exponential growth model $A = A_0 e^{kt}$, in which t is the number of years after 2000, to find an exponential growth function that models the data.

b. In which year will the Palestinian population be 9 million?

Example 2: A bird species in danger of extinction has a population that is decreasing exponentially ($A = A_0 e^{kt}$). Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

Example 3: Use the exponential growth model, $A = A_0 e^{kt}$, to show that the time it takes for a population to triple is given by $t = \frac{\ln 3}{k}$.

Example 4: The August 1978 issue of *National Geographic* described the 1964 find of bones of a newly discovered dinosaur weighing 170 pounds, measuring 9 feet, with a 6 inch claw on one toe of each hind foot. The age of the dinosaur was estimated using potassium-40 dating of rocks surrounding the bones.

a. Potassium-40 decays exponentially with a half-life of approximately 1.31 billion years. Use the fact that after 1.31 billion years a given amount of Potassium-40 will have decayed to half the original amount to show that the decay model for Potassium-40 is given by $A = A_0 e^{-0.52912t}$, where t is in billions of years.

b. Analysis of the rocks surrounding the dinosaur bones indicated that 94.5% of the original amount of Potassium-40 was still present. Let $A = 0.945A_0$ in the model in part (a) and estimate the age of the bones of the dinosaur.

EXPRESSING AN EXPONENTIAL MODEL IN BASE *e*

___________________ is equivalent to ____________________

Example 5: Rewrite the equation in terms of base *e*. Express the answer in terms of a natural logarithm and then round to three decimal places.

a. $y = 1000(7.3)^x$

b. $y = 4.5(0.6)^x$

Section 13.1: THE CIRCLE

When you are done with your 13.1 homework you should be able to…

- π Write the standard form of a circle's equation
- π Give the center and radius of a circle whose equation is in standard form
- π Convert the general form of a circle's equation to standard form

Warm-up:

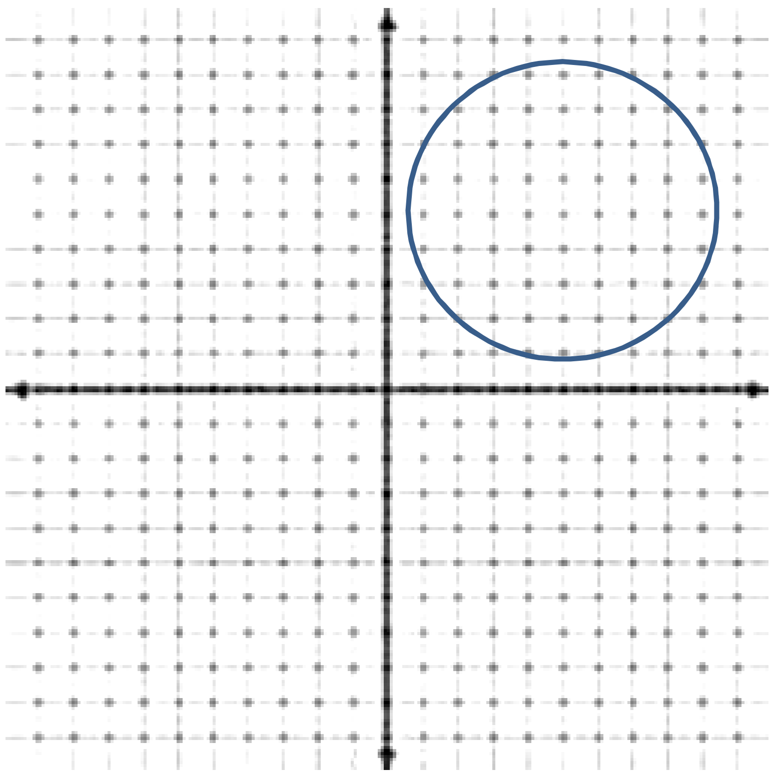
1. Solve by completing the square.

$2x^2 - 6x + 2 = 3$

2. Identify the vertex of the quadratic function $f(x) = -(x+4)^2 + 1$

DEFINITION OF A CIRCLE

A circle is the ______ of all points in a plane that are ____________________ from a __________ point, called the ________________ The fixed distance from the circle's ________________ to any point on the _________________ is called the ______________.



THE STANDARD FORM OF THE EQUATION OF A CIRCLE

The standard form of the equation of a circle with center ____________ and radius ______ is

Example 1: Write the standard form of the equation of the circle with the given center and radius.

a. Center: $(0,0)$, $r=8$

b. Center: $(1,-6)$, $r=\sqrt{2}$

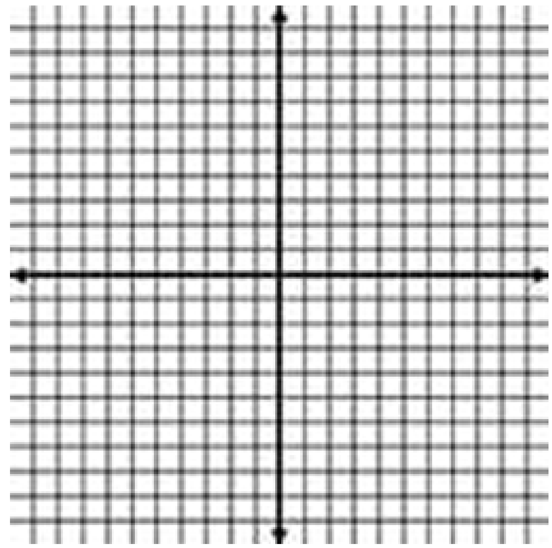
Center: $\left(-\frac{1}{2},0\right)$, $r=10$

THE GENERAL FORM OF THE EQUATION OF A CIRCLE

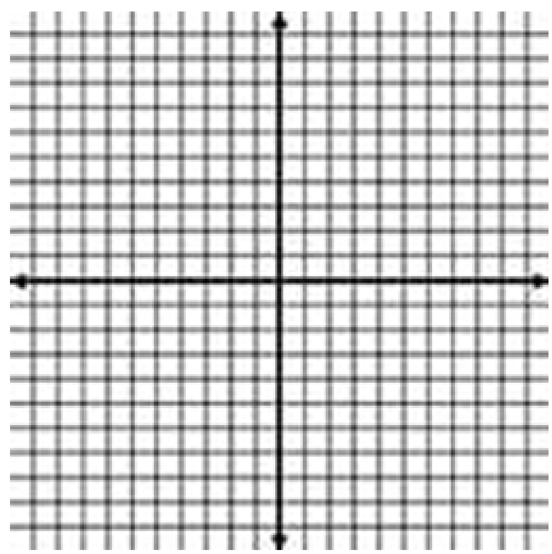
The general form of the equation of a circle with center ____________ and radius ______ is

Example 2: Write the equation of the circle in standard form, if necessary. Then give the center and radius of each circle and graph the equation.

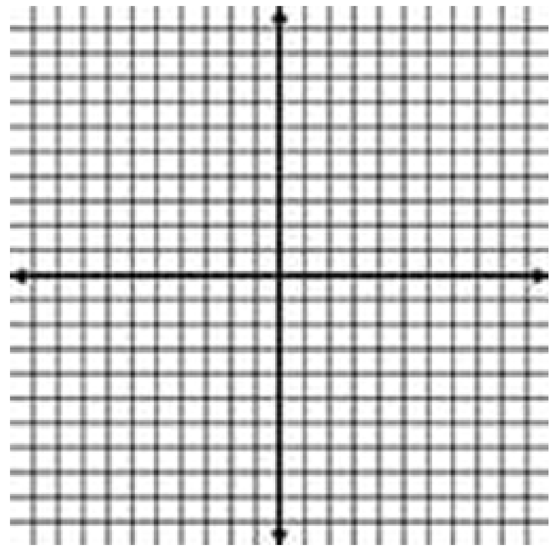
a. $x^2+(y-1)^2=16$



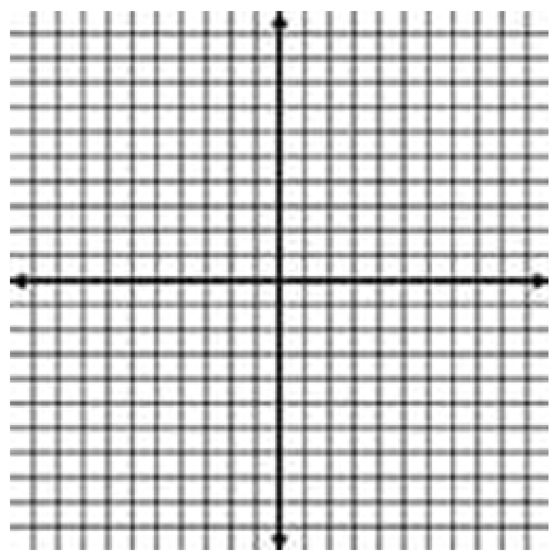
b. $x^2+y^2+8x+4y+16=0$



c. $x^2 + y^2 - 6x - 7 = 0$



d. $x^2 + y^2 - 49 = 0$



Section 13.5: SYSTEMS OF NONLINEAR EQUATIONS IN TWO VARIABLES

When you are done with your 13.5 homework you should be able to…

- π Recognize systems of nonlinear equations in two variables
- π Solve systems of nonlinear equations by substitution
- π Solve systems of nonlinear equations by addition
- π Solve problems using systems of nonlinear equations

WARM-UP:

1. Solve the system by the substitution method.

$$\begin{cases} x + y = 6 \\ 4x - y = 4 \end{cases}$$

2. Solve the system by the addition method.

$$\begin{cases} 2x - 4y = 3 \\ x = 2y + 4 \end{cases}$$

A _________________ of two _________________ equations in two variables, also called a _____________________ system, contains at least one equation that cannot be expressed in the form ______________________. A ___________________ of a nonlinear system in two variables is an ordered pair of real numbers that satisfies all equations in the ___________________. The solution ___________ of the system is the set of all such ordered pairs. As with linear systems in two variables, the solution of a nonlinear system (if there is one) corresponds to the ______________________ point(s) of the _______________ of the equations in the system.

Example 1: Solve each system by the substitution method.

a.

$$\begin{cases} x - y = -1 \\ y = x^2 + 1 \end{cases}$$

b.

$$\begin{cases} y = x^2 + 4x + 5 \\ y = x^2 + 2x - 1 \end{cases}$$

c.

$$\begin{cases} xy = -12 \\ x - 2y + 14 = 0 \end{cases}$$

Example 2: Solve each system by the addition method.

a.

$$\begin{cases} 4x^2 - y^2 = 4 \\ 4x^2 + y^2 = 4 \end{cases}$$

b.

$$\begin{cases} x^2 - 2y = 8 \\ x^2 + y^2 = 16 \end{cases}$$

c.

$$\begin{cases} x^2 + y^2 = 4 \\ x^2 + (y-3)^2 = 9 \end{cases}$$

Example 3: The difference between the squares of two numbers is 5. Twice the square of the second number subtracted from three times the square of the first number is 19. Find the numbers.

Example 4: Find the length and width of a rectangle whose perimeter is 40 feet and whose area is 96 square feet.